Continuity of a function at a point.
By Definition, f(x) is continuous at x = c iff all of the following are true:

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

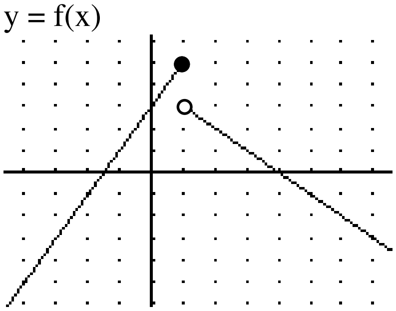
Try the following on your own:

1. Draw a graph where (1) is true, but (2) and (3) are not
2. Draw a graph where (2) is true, but (1) and (3) are not
3. Draw a graph where (1) and (2) are true, but (3) is not
4. Draw a graph where (3) is true, but not (1) or (2)

By definition, f(x) has a **Removable Discontinuity** at x = c if by creating a new or different value for x = c, the function would be continuous at c.

Otherwise a function that is not continuous at x = c has a **Non-Removable Discontinuity** at x = c.

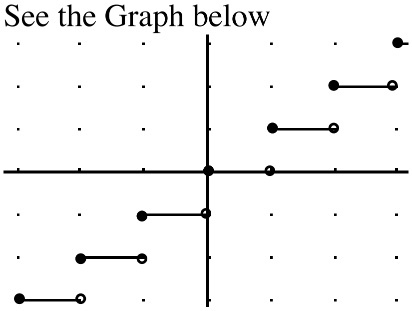
y = f(x)



f(1) = 5, $\lim_{x \to 1}$ f(x) Does Not Exist □ Non-Removable Discontinuity at x = 1

f(x) = [|x|] is called the **Greatest Integer Function**. It is often referred to as a **Step Function**:

See the Graph below



f(2.5) = 2, f(3) = 3, f(0) = 0, f(-2.7) = -3

$\lim_{x \to 1.5}$ f(x) = 1, $\lim_{x \to 2^-}$ f(x) = 1, $\lim_{x \to 2^+}$ f(x) = 2

In our class, we will accept that the Following Functions are Continuous at every point **in their domain**:

1. Polynomial Functions
2. Rational Functions
3. Radical Functions
4. Trigonometric Functions

Discus the continuity of the following piecewise function.

$$f(x) = \begin{cases} 2x + 1 & \text{if } x > 0 \\ 3x - 3 & \text{if } x \le 0 \end{cases}$$

1. $f(0) = -3$. Therefore the function is defined at 0.
2. $\lim_{x \to 0} f(x)$ Does Not Exist
3. $\lim_{x \to 0} f(x)$ cannot equal $f(0)$

The Function **is not continuous at x = 0**,
The Function **is continuous for all x ≠ 0** because these values give polynomials.

Discus the continuity of the following piecewise function.

$$f(x) = \begin{cases} x^2 + 3x & \text{if } x \le 1 \\ 5x - 1 & \text{if } x > 1 \end{cases}$$

1. $f(1) = 4$. Therefore the function is defined at $x = 1$.
2. $\lim_{x \to 1} f(x) = 4$
3. $\lim_{x \to 1} f(x) = f(1) = 4$

The Function **is continuous at x = 1**.
The Function **is continuous for all x ≠ 1** because these values give polynomials.
The Funcion is **everywhere continuous**.

When viewing the assignment, recall that:

.϶. Means “Such That”

∈ Means “Is An Element Belonging To”

Also recall that when a Domain is not specifically stated, the domain will be all “Legal Real Numbers”.

PC Assignment 257 Wednesday, June 3, 2015 Hour      Name ______________________

Page 1 - Discuss the Continuity for each.

1. $g(x) = \sqrt{25 - x^2}$ .э. $x \in [-5, 5]$

2. $f(x) = \begin{cases} 3 - x, & x \le 0 \\ 3 + x/2, & x > 0 \end{cases}$ .э. $x \in [-1,4]$

3. $g(x) = \frac{1}{x^2 - 4}$ .э. $x \in [-1, 2]$

4. $f(x) = x^2 - 2x + 1$

5. $f(x) = \frac{x - 3}{x^2 - 9}$

6. $f(x) = \frac{|x + 2|}{x + 2}$

7. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

8. $f(x) = \tan x$ .э. $x \in [0, \pi]$

9. Find the value for *a* that makes f(x) continuous on all real numbers.
$f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$

10. On the Polar - Plane, Sketch a graph of the Polar Equation: $r = 2 + 2\cos 2\theta$

11. On the Polar - Plane, Sketch a graph of the Polar Equation: $r = 2 + \cos 2\theta$

12. Use the Difference Quotient to find $\frac{d}{dx}(\sin x)$