

1. When we find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, we are finding the **Derivative** of $f(x)$.
2. For example, we have found that the derivative of $f(x) = \sin x$, is $\cos x$.
3. We should know that the derivative of a position function is the corresponding velocity function.
4. We should also know that the derivative of a velocity function is the corresponding acceleration function.
5. The position of an object is $s(t) = t^2 + t$.

- a. Find the position at $t = 3$.

$$s(3) = 9 + 3 = 12$$

- b. Find the position at $t = 5$.

$$s(5) = 25 + 5 = 30$$

- c. Find the average velocity from $t = 3$ to $t = 5$

$$\frac{f(5) - f(3)}{5 - 3} = \frac{30 - 12}{2} = \frac{18}{2} = 9$$

- d. Find the instantaneous velocity at $t = 3$.

$$\begin{aligned} v(t) &= \lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(t+h)^2 + (t+h) - (t^2 + t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{t^2 + 2th + h^2 + t + h - t^2 - t}{h} \\ &= \lim_{h \rightarrow 0} \frac{2th + h^2 + h}{h} \\ &= \lim_{h \rightarrow 0} (2t + h + 1) \\ &= 2t + 1 \end{aligned}$$

$$\text{Therefore } v(3) = 6 + 1 = 7$$

6. Find the derivative of $f(x) = ax^3$, where a is a constant.

$$\begin{aligned} &\lim_{h \rightarrow 0} \frac{a(x+h)^3 - ax}{h} \\ &= \lim_{h \rightarrow 0} \frac{a(x^3 + 3x^2h + 3xh^2 + h^3) - ax}{h} \\ &= \lim_{h \rightarrow 0} \frac{3ax^2h + 3axh^2 + ah^3}{h} \\ &= \lim_{h \rightarrow 0} (3ax^2 + 3axh + ah^2) \\ &= 3ax \end{aligned}$$

7. $f(x) = ax^n$. The expressions $f'(x)$ or $\frac{d[f(x)]}{dx}$ or $D_x[f(x)]$ all refer to the Derivative of $f(x)$.

8. $f(x) = ax^n$. Find $f'(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{a(x+h)^n - ax^n}{h} = \lim_{h \rightarrow 0} \frac{a(x^n + nx^{n-1}h + \dots) - ax^n}{h} = \lim_{h \rightarrow 0} \frac{nx^{n-1}h + \dots}{h} = nx^{n-1}$$

9. Notice that in #8, we can now take the exponent on the variable and multiply the constant by it, then reduce the original exponent by one.

10. $f(x) = 3x^4 - 2x^3 + 2x^2 - 3x + 4$. Using the discovery mentioned in #9, find $f'(x)$.

$$\boxed{f'(x) = 12x^3 - 6x^2 + 4x - 3}. \text{ We took the derivative of each term.}$$

11. Find the anti-derivative of $f(x) = 18x^5 - 20x^3 + 7$.

Answer: $3x^6 - 5x^4 + 7x + C$, where C is any Constant. If you want to check, plug any number in for C , and take the derivative and see if you get the original $f(x)$ back.

12. Since acceleration is the derivative of velocity, and velocity is the derivative of position, then we can now say that velocity is the anti-derivative of acceleration, and position is the anti-derivative of velocity.

13. Given velocity, $v(t) = 3t^2 + 6t - 2$, and position $s(t)$ is 9 when $t = 2$. Find the position $s(3)$.

$$s(t) = t^3 + 3t^2 - 2t + C, \text{ but we know that } s(2) = 9 = 8 + 12 - 4 + C = 9 \rightarrow C + 16 = 9 \rightarrow C = -7$$

$$s(t) = t^3 + 3t^2 - 2t - 7. \text{ Therefore } s(3) = 27 + 27 - 6 - 7 = 41.$$

1. An object is moving at a velocity of $v(t) = 21t^2 + 30$ meters per second. Find the Average velocity on the time interval $[2, 4]$.
2. Using the information given in #1 and our new ways of finding a derivative, find the exact acceleration at time, $t = 3$ seconds.
3. An object has position $s(t) = \sin(t)$. Find the average velocity on the time interval $\left[0, \frac{\pi}{3}\right]$.
4. Using the information given in #3, find the exact acceleration at time $= \frac{\pi}{3}$.
5. An object has acceleration of an object is $a(t) = 2t - 4$, and $v(5) = 12$. Find $v(2)$.
6. $f(x) = 3x^2 - 2x + 1$. Write the point-slope equation of the tangent line to $f(x)$ at $x = 3$.
7. $f(x) = \sin x + 6x^2 + x$. Find $f'(x)$.