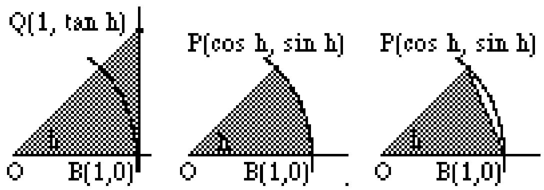
Special Limits

1. Suppose $f(x) \le g(x) \le h(x)$. Also Suppose $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} h(x) = L$ .
2. What would you think of $\lim_{x \to c} g(x)$ ?
3. Of course $\lim_{x \to c} g(x) = L$ also. This **logic** is known as "**The Squeeze Theorem**", or "**The Sandwich Theorem**".
4. Below are portions of 3 unit circles with the same central angle, h, and different shaded regions.




5. The areas of the shaded regions from large to small are $\frac{\tan h}{2} \ge \frac{h}{2} \ge \frac{\sin h}{2}$.
6. Multiply the inequalities by $\frac{2}{\sin h}$, and we get: $\frac{1}{\cos h} \ge \frac{h}{\sin h} \ge 1$
7. Now take the reciprocal of each part and get: $\cos h \le \frac{\sin h}{h} \le 1$
8. Now take the limit of the left and right sides as h goes to 0:
   $\lim_{h \to 0} \cos h = 1$ and $\lim_{h \to 0} 1 = 1$
9. By the Squeeze Therom, $\boxed{\lim_{h \to 0} \frac{\sin h}{h} = 1}$ also. This is a **very important limit** to remember.
10. Find $\lim_{x \to 0} \frac{1 - \cos x}{x}$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = \lim_{x \to 0} \frac{1 - \cos x}{x} \bullet \frac{1 + \cos x}{1 + \cos x} = \lim_{x \to 0} \frac{1 - \cos^2 x}{x(1 + \cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{x(1 + \cos x)}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \bullet \frac{\sin x}{1 + \cos x} = 1 \bullet 0 = 0.$$

11. So now we have a 2nd very important limit: $\boxed{\lim_{x \to 0} \frac{1 - \cos x}{x} = 0}$.

Pre-Calculus 2 Assignment 252 Wednesday, May 27, 2015 Hour Name

1. $\lim_{x \to 2} \frac{2-x}{x^2-4} =$

2. $\lim_{x \to 0} \frac{\tan x}{x} =$

3. $\lim_{x \to 0} \frac{\sin 4x}{x} = 4$ Explain why this is true.

Exer. 4-11: Find: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

4. $f(x) = 5 - 6x$

5. $f(x) = \sqrt{x}$

6. $f(x) = \sqrt{x-2}$

7. $f(x) = \sin x$

8. $f(x) = \frac{1}{x}$

9. $f(x) = 4 - 2x - x^2$

10. $f(x) = \frac{1}{x+2}$

11. $f(x) = \frac{1}{x-1}$