Tangent Line to a Curve at a Point

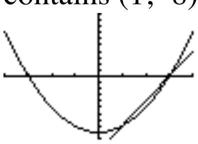
1. A line is tangent to a curve at a point when two conditions exist.
   a. The line and the curve must share that point
   b. The line and the curve have the same slope at that point

2. So when we talk about the slope of a curve at a point, we are also talking about the slope of the tangent line at that point.

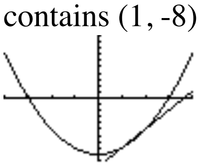
3. Consider the graph of $f(x) = x^2 - 9$

4. We want to find the **slope** of f(x) at x = 1. This means that we want to find the slope of the line **tangent** to the graph at the point (1, -8).

5. Observe the line containing (1, -8) and (3, 0), where the slope is 4. This is one of many **secant lines** that contains (1, -8).



6. Also observe the line containing (1, -8) and (2, -5), where the slope is 3. This is another secant line that contains (1, -8).



7. Although this new line is closer to being tangent to f(x) at the point (1, -8), it is not exactly tangent.

8. To get an exact tangent, we need to take the limit of the slope of a secant line as that second point gets closer to the (1, -8).

9. We will pick a very small number, h, so 1 + h is very close to, but not equal to, 1.

10. f(1 + h) is very close to, but not equal to -8.

11. So now we have two points: (1, f(1)) and (1+h, f(1+h)).

12. The slope of the secant line containing those two points is: $\frac{f(1+h)-f(1)}{(1+h)-1} = \frac{f(1+h)-f(1)}{h}$

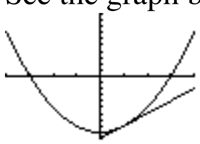
13. Of course, you will notice that this is the difference quotient that we have seen in the past.

14. The exact slope of the tangent line will be found by computing: $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

15. $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)^2 - 9 - (1^2 - 9)}{h} = \lim_{h \to 0} \frac{1 + 2h + h^2 - 9 - 1 + 9}{h}$
$= \lim_{h \to 0} \frac{2h + h^2}{h} = \lim_{h \to 0} \frac{h(2+h)}{h} = \lim_{h \to 0} (2 + h) = 2.$

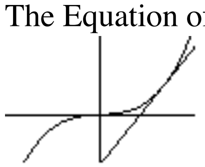
16. Now that we know that the slope of the tangent line at x = 1 is 2, along with the point of tangency being (1, -8), we can now write the equation of that line.
Point-Slope Form: $\boxed{y + 8 = 2(x - 1)}$ or Slope-Intercept Form: $\boxed{y = 2x - 10}$.

17. See the graph below.



18. Write the equation of the tangent line to $f(x) = 2x^3 + x$ at x = 2.
The Point is $\boxed{(2, 18)}$.
The Slope is $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{2(2+h)^3 + (2+h) - (2 \cdot 2^3 + 2)}{h}$
$= \lim_{h \to 0} \frac{2(8 + 12h + 6h^2 + h^3) + 2 + h - 16 - 2}{h} = \lim_{h \to 0} \frac{16 + 24h + 12h^2 + 2h^3 + h - 16}{h}$
$= \lim_{h \to 0} \frac{25h + 12h^2 + 2h^3}{h} = \lim_{h \to 0} \frac{h(25 + 12h + 2h^2)}{h} = \lim_{h \to 0} (25 + 12h + 2h^2) = \boxed{25}$.

19. The Equation of the tangent line: Point-Slope $\boxed{y - 18 = 25(x - 2)}$ or Slope-Intercept $\boxed{y = 25x - 32}$.



Pre-Calculus Assignment 251 Tuesday, May 26, 2015 Hour Name

Find the point-slope equation of the tangent line to the given function at the given value of x.

1. $f(x) = 2x^2 - 2,\ x = 5$

2. $f(x) = x^3 + 8,\ x = 1$

3. $f(x) = \sqrt{x},\ x = 16$

4. $f(x) = x^2 + 2x - 1,\ x = 5$

5. $f(x) = -3x^2 + 2x,\ x = 3$

6. $f(x) = 2x^3 - x^2,\ x = -2$

7. $f(x) = \sqrt{x+7},\ x = 2$