1. Why this sudden interest in situations that appear like $\frac{0}{0}$? Most people, when asked to describe or define velocity will give the fraction like $\frac{s}{t}$, where s represents distance and t represents time.
2. Consider driving from Royal Oak to Chicago, a distance of 280 miles. Let's also say that you left Royal Oak at 3:00pm EST and arrived in Chicago at 7:00pm EST. How fast were you driving?.
3. You probably did not get 65 mph, nor were you likely to get 30 mph, and yet those velocities were likely arrived at during the trip. You probably got 70 mph.
4. But, that's not the correct answer. Actually you were going at a variety of velocities. You were probably stopped by a traffic light for a while, going 0 mph. At another point in time, you were going 40 mph.
5. Let's say you wanted to compute your exact velocity while moving along the freeway at exactly 4:55 pm. This is a big problem if you revert back to $\frac{s}{t}$, where s represents distance and t represents time. How far did you travel at that instant? How much time passed in that instant? We see that this produces a $\frac{0}{0}$ situation. We know that there must be an actual answer because we are moving, but how do we find it?
6. In this question, we are talking about "**instantaneous velocity**", not "**average velocity**".
7. To find instantaneous velocity, we need to use a limit.
8. Let's say that we are traveling in such a way that our distance from a reference point is given by the following function: $s(t) = t^2 + 3t + 1$. Find the velocity at t = 3.
9. We know that at time t = 3, the position is $s(3) = (3)^2 + 3(3) + 1$
10. Let h be a very small amount of time so that 3 + h is very close to 3. At this new moment in time, the position will be $s(3 + h) = (3 + h)^2 + 3(3 + h) + 1$
11. Now we have distance traveled: $s(3 + h) - s(3)$.
12. We also have a small passage of time: $(3 + h) - 3 = h$
13. Remember our old **Difference Quotient**? As we try to find the instantaneous velocity at time t = 3, we can try using our old version of $\frac{s}{t}$ which will give: $\frac{s(3+h)-s(3)}{h}$, but this doesn't help.
14. But, if we apply the limit, it just might work. $\lim_{h \to 0} \frac{s(3+h)-s(3)}{h}$

$$= \lim_{h \to 0} \frac{(3+h)^2 + 3(3+h) + 1 - \left[(3)^2 + 3(3) + 1\right]}{h} = \lim_{h \to 0} \frac{(9+6h+h^2) + (9+3h) + 1 - \left[(9) + (9) + 1\right]}{h}$$

$$= \lim_{h \to 0} \frac{9h + h^2}{h} = \lim_{h \to 0} \frac{h(9+h)}{h} = \lim_{h \to 0} (9 + h) = 9 .$$

15. So … At time t = 3, the exact velocity at time t = 3 is v(3) = 9.
16. Finally, a real-life type of an application of the limit.

Pre-Calc 2 Assignment 250 Friday, May 22, 2015 Hour Name ______________________________

Finding Limits of Difference Quotients

1. f(x) = 3x – 5, Find $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$.

2. f(x) = $2x^2 + 3$, Find $\lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$.

3. f(x) = $-3x^2 + 3x$, Find $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$.

4. f(x) = $5x^2 - 2x + 2$, Find $\lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$.

5. f(x) = $x^3 - 4x$, Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.