

Typical ways to evaluate limits of fractions analytically when direct substitution fails:

1. $\lim_{x \rightarrow 5} (3x + 2)$ Translates to “The Limit as x goes to 5 of $3x + 2$ ”

This means that as x gets closer to 5 from below and above, find what $(3x + 2)$ is approaching.

Consider the table:

x	4.6	4.7	4.8	4.9	5.1	5.2	5.3	5.4
$3x + 2$	15.8	16.1	16.4	16.7	17.3	17.6	17.9	18.2

Notice as x gets closer to 5 from either side right or left, $3x + 2$ gets closer to 17.

2. Therefore $\lim_{x \rightarrow 5} (3x + 2) = 17$

3. If you simply plug in 5 for x , you will get 17, this is called **Direct Substitution**

4. The problem is that Direct Substitution does not always work.

5. Consider $\lim_{x \rightarrow 0} \frac{\sin x}{x}$. Direct Substitution will lead to $\frac{0}{0}$, but that is NOT the answer.

6. Consider the table:

x	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.4	0.5
$(\sin x) / x$	0.95885	0.97355	0.98507	0.99335	0.99833	0.99833	0.99355	0.99507	0.97355	0.958885

Notice that as x get closer to 0 from either side right or left, $\frac{\sin x}{x}$ gets closer to 1

7. Therefore $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

8. We now see a scenario where there is a consideration of a $\frac{0}{0}$ situation that leads to a definite answer.

9. Consider $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2x^2 - 5x + 2}$. Again Direct substitution will not work.

10. Factoring can lead to an answer: $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2x^2 - 5x + 2} = \lim_{x \rightarrow 2} \frac{(x + 3)(x - 2)}{(2x - 1)(x - 2)} = \lim_{x \rightarrow 2} \frac{x + 3}{2x - 1} = \frac{5}{3}$

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1.
$$\lim_{x \rightarrow 6} \frac{x-6}{x^2-36} =$$

2.
$$\lim_{x \rightarrow 9} \frac{9-x}{x^2-81} =$$

3.
$$\lim_{x \rightarrow -1} \frac{1-2x-3x^2}{1+x} =$$

4.
$$\lim_{x \rightarrow -4} \frac{2x^2+7x-4}{x+4} =$$

5.
$$\lim_{x \rightarrow 2} \frac{x^3-8}{x-2} =$$

6.
$$\lim_{x \rightarrow -1} \frac{x^3+2x^2-x-2}{x^3+4x^2-x-4} =$$