

## Parametrics

$x = t^2 + 1$ ,  $y = 2t$  make up a pair of Parametric Equations. The parameter is  $t$ .

Suppose  $t$  represents time. At  $t = 0$ , an object would be at position  $(1, 0)$ . At  $t = 2$ , an object would be at position  $(5, 4)$ . At  $t = 3$ , an object would be at position  $(10, 6)$ . From these calculations, we can see that the object is moving along a predictable path that has a specific orientation.

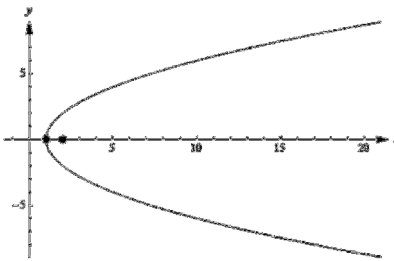
Write an equation for the above relation exclusive in terms of  $x$  and  $y$ . In other words, eliminate the parameter.

1. Solve for  $t$  in one equation

$$t = \frac{1}{2}y$$

2. Substitute into the other equation

$$x = \left(\frac{1}{2}y\right)^2 + 1 \quad \text{This is a parabola with standard form: } y^2 = 4(x - 1)$$



The above graph alone does not depict the orientation of travel. However, the changing of positions over time calculated from the parametric equations shows that the orientation of travel is along an upward path along the parabola.

## Polar vs Rectangular

Recall that:  $x = r \cos \theta$ ,  $y = r \sin \theta$ ,  $r = \sqrt{x^2 + y^2}$ , and  $\theta = \arctan\left(\frac{y}{x}\right)$ . We can use these equations to

convert polar equations to rectangular and from rectangular equations to polar.

1.  $r \cos \theta = x$  Convert to Rectangular.

$$x = x^2 + y^2$$

2.  $r^2 \sin \theta = y$  Convert to Rectangular.

$$\text{Multiplying both sides by } r, \text{ we get: } r^2(r \sin \theta) = r y \rightarrow (x^2 + y^2)y = x$$

3. Convert the point in polar form  $\left(\frac{5\pi}{4}, -4\right)$  to rectangular form.

$\theta = \frac{5\pi}{4}$  points into quadrant 3, but with  $r = -4$ , we must look into the opposite direction into quadrant 1.

Therefore the rectangular point is at  $\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$

1.  $x = t - 2, y = 2t + 3$  Write an equation only in  $x$  and  $y$ .

2.  $x = 1 - 2t, y = 1 + t$  Write an equation only in  $x$  and  $y$ .

3.  $x = t^3, y = t^2$  Write an equation only in  $x$  and  $y$ .

4.  $(y - 2)^2 = -16(x + 3)$  Write the focus and the equation of the directrix.

5.  $\frac{(x+3)^2}{289} + \frac{(y-10)^2}{64} = 1$  Find the foci and the equation of the directrix and eccentricity.

6.  $\frac{(x-7)^2}{16} - \frac{(y-1)^2}{9} = 1$  Find the foci, the directrices, asymptotes, and eccentricity.

7.  $r^2 + r = 3 \sin \theta$  Convert to Rectangular

8.  $x^2 + y^2 - 6y = 9$  Convert to Polar