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A **Hyperbola** is the set of all points (x, y) in a plane, the difference of whose distances from two distinct fixed points, the **foci**, is a positive constant.

The graph of a hyperbola has two disconnected parts called the **branches**. The segment through the foci is the **Transverse Axis**. The mid-point of the Transverse Axis is the **Center** of the hyperbola.

The derivation of the equation for the hyperbola is similar to that of the ellipse. However, the distances from a point to each focus is not added, but they are subtracted to give a constant. The relationship between a, b, and c is also different in that  $a^2 + b^2 = c^2$ . In this case, it is c that is the larger of the two.

a = distance from center to each vertex = length of semi-transverse axis

b = length of the semi-conjugate axis

c = distance from center to each focus

The Standard Form Equation of a Hyperbola with center (h, k) is:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
 When the Transverse axis is horizontal.  
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
 When the Transverse axis is vertical.

The figure below is a graph for  $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{9} = 1$  The dimensions of the rectangle is 2a by 2b There are asymptotes that pass through the center (2, 3) with slopes  $\pm \frac{3}{4} = \pm \frac{b}{a}$  since the transverse axis is horizontal. If the transverse axis is vertical, the slopes would be  $\pm \frac{4}{3} = \pm \frac{a}{b}$ .



We have h = 2, k = 3, a = 4, b = 3, and c = 5

The center is at (2, 3). Vertices are at (-2, 3) & (6, 3). The Foci are at  $(2 \pm 5, 3)$ The equations for the asymptotes above in point slope form are:

$$y-3 = \frac{3}{4}(x-2)$$
 and  $y-3 = -\frac{3}{4}(x-2)$ 

Exer. 1-6: Find the vertices, the foci, and asymptotes of the hyperbola.



Exer. 8-16: From the Given Information, write the Equation of the Hyperbola in Standard Form

7. V(-2, -2), (-2, -4); F(-2, -1), (-2, -5)	
8. V(0, 2), (2, 2); F(-2, 2), (4, 2)	
9. $F(0, \pm 5)$ ; Conjugate axis is 4 units	
10. V(±4, 0); Contains (8, 2)	
11. V( $\pm 3, 0$ ); Asym: $y = \pm 2x$	
12. F(0, ±10); Asym: $y = \pm \frac{1}{3} x$	
13. x-int: $\pm 5$ ; Asym: $y = \pm 2x$	
14. y-int: $\pm 2$ ; Asym: $y = \pm \frac{1}{4} x$	
15. Vert Tans Axis = 10; Conj Axis = 14	
16. Horiz Trans Axis = 6; Conj Axis = 2	