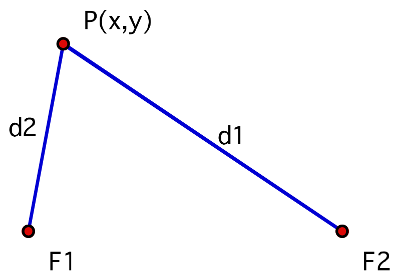
241 - Definition of Ellipse:

Given 2 points in a plane. The set of all points in that plane whose respective distances to those 2 points is a fixed sum forms an ellipse.

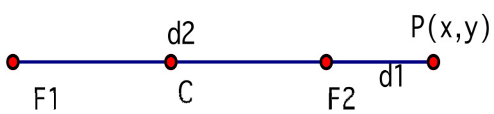
In these examples: $d_1 + d_2$ is a constant sum.

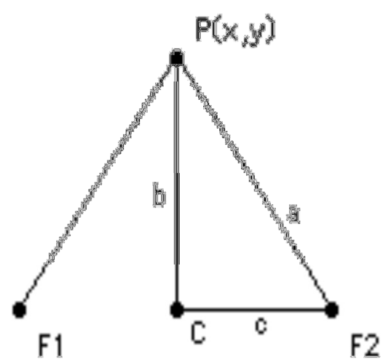


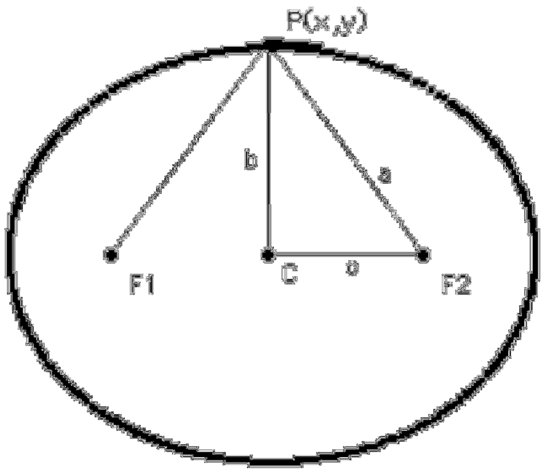
If we choose one point for P so that it is equidistant from F1 and F1

So d1 = d2

Let's call each of those lengths a







Notice that $c^2 + b^2 = a^2$ therefore $a^2 - c^2 = b^2$

1. $F_1(-c, 0)$ $C(0, 0)$ $F2((c, 0)$

2. $F_1P = F_2P = a$

3. $F_1P + F_2P = 2a$

4. $\sqrt{(x + c)^2 + y^2} + \sqrt{(x - c)^2 + y^2} = 2a$

5. $\sqrt{(x + c)^2 + y^2} = 2a - \sqrt{(x - c)^2 + y^2}$

6. $(x + c)^2 + y^2 = 4a^2 - 4a\sqrt{(x - c)^2 + y^2} + (x - c)^2 + y^2$

7. $4a\sqrt{(x - c)^2 + y^2} = 4a^2 + (x^2 - 2cx + c^2) - (x^2 + 2cx + c^2)$

8. $4a\sqrt{(x - c)^2 + y^2} = 4a^2 - 4cx$

9. $a\sqrt{(x - c)^2 + y^2} = a^2 - cx$

10. $a^2[(x - c)^2 + y^2] = a^4 - 2a^2cx + c^2x^2$

11. $a^2(x^2 - 2cx + c^2 + y^2) = a^4 - 2a^2cx + c^2x^2$

12. $a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2 = a^4 - 2a^2cx + c^2x^2$

13. $a^2x^2 + a^2c^2 + a^2y^2 = a^4 + c^2x^2$

14. $a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2$

15. $(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$

16. $b^2x^2 + a^2y^2 = a^2b^2$

17. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

18. a = Distance from Center to each Major Vertex

19. b = Distance from Center to each Minor Vertex

20. c = Distance from Center to each Focus

21. $a > b$ and $a > c$

Example Problems:

1. $\frac{x^2}{25} + \frac{y^2}{9} = 1$

1a. Find the Coordinates of the Major Vertices
(±5, 0)

1b. Find the Coordinates of the Minor Vertices
(0, ±3)

1c. Find the Coordinates of the Foci
(±4, 0)

2. $\frac{(x - 3)^2}{25} + \frac{(y + 1)^2}{9} = 1$

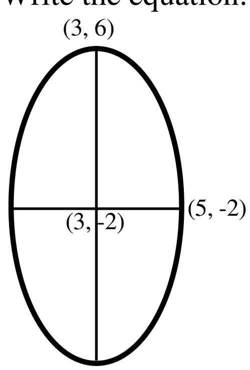
2a. Find the Center
(3, -1)

2b. Find the Coordinates of the Major Vertices
(-2, -1) and (8, -1)

2c. Find the Coordinates of the Minor Vertices
(3, -4) and (3, 2)

2d. Find the Coordinates of the Foci
(-1, -1) and (7, -1)

3a. Write the equation:



a = 8
b = 2
$\frac{(x - 3)^2}{4} + \frac{(y + 2)^2}{64} = 1$

3b. Find the coordinates of the foci
$c^2 = a^2 - b^2 = 64 - 4 = 60$
$c = \sqrt{60} = 2\sqrt{15}$
Foci: $(3, -2 - 2\sqrt{15})$ and $(3, -2 + 2\sqrt{15})$