239

In how many ways can 6 people stand around a campfire?

At first glance you might think 6!, but let's consider this more closely.

A-B-C-D-E-F = B-C-D-E-F-A = C-D-E-F-A-B = D-E-F-A-B-C = E-F-A-B-C-D = F-A-B-C-D-E

This is an example the above arrangements are all the same because of the circular nature of the arrangements.

So we take 6! and divide by 6 and get 5! = $\boxed{120}$

How many ways can 4 keys be arranged on a small straight bar?

Again at first glance, you might think 4!, but let's look more closely.

A-B-C-D = D-C-B-A

This is because we can turn the bar around without changing the order of the keys relative to each other.

So we get 4!/2 = $\boxed{12}$

In how many ways can 7 charms be arranged on a charm bracelet with no clasp?

This is circular, but also invertible. So we compute $\frac{7!}{7\cdot 2} = \frac{6!}{2} = \boxed{360}$

In how many ways can 7 charms be arranged on a charm bracelet with a clasp past which the charms cannot move?

At first this seems circular, but it is not, because the charms are placed relative to the clasp. However this is an invertible situation. So we compute $\frac{7!}{2} = \boxed{2520}$

In conclusion:

If arrangements are truly circular, divide the initial factorial by the number of items.

If arrangements are invertible, divide the initial factorial by 2.

If arrangements are both circular and invertible, divide the initial factorial by the number of items and also divide by 2.

Pre-Calculus Assignment 239 Friday, April 24, 2015 Hour Name ________________________________

Do the Work on a Separate Sheet of Paper

1. $(2x^2 + 3y)^7$ Find the 3rd term.
2. How many arrangements are possible given 7 charms on a bracelet with no clasp?
3. In how many ways can 6 people line up at an ice cream stand?
4. A class consists of 19 students but only 8 are present. A pass comes for a student. What are the odds that that student is not present?
5. A coin and 2 dice are tossed. Find the probability of getting a head and a 7.
6. The odds of Kevin annoying Karen on any given day are 5 : 9. Find the probability that Kevin will annoy Karen today.
7. $5 + 8 + 11 + 14 + 17 + \ldots + 1376 =$
8. The configuration of a license plate is 3 letters followed by 4 digits. How many variations are possible if the middle letter is not a vowel (treat y is a vowel) and the first digit is not zero?
9. How many distinguishable arrangements of the letters in mississauga are there?
10. Using a standard deck of 52 cards, find the probability of having a 5-card hand with exactly 2 aces and exactly 1 king.
11. A test has 30 True-False questions. Grant answers each question randomly. In how many ways can he get 12 correct?
12. Our classroom has 30 desks and 19 students. How many possible seating charts can be made?
13. $2048 + 512 + 128 + 32 + 8 + 2 + \ldots =$
14. In how many ways can 6 charms be arranged on a charm bracelet?
15. In how many ways can 8 people be arranged in a circle?
16. From a group of 20 boys and 15 girls, in how many ways can a debate team of 3 boys and 3 girls be chosen?
17. From a group of 20 boys and 15 girls, in how many ways can we put 3 boys and 3 girls in 6 adjacent chairs if the girls must sit next to each other and the boys must sit next to each other?
18. From a hurdle race of 8 girls, the top 3 place winners are on the awards stand with designated positions for 1^{st}, 2^{nd}, and 3^{rd}. How many possible arrangements are there on the awards stand?
19. 5, 10, 20, 40, 80, … Write the generator for the sequence.
20. Using Math Induction, Prove: $\sum_{i=1}^{n}(3i+4) = \frac{n(3n+11)}{2}$