

Counting Principles

Ex.1: Eight pieces of paper, numbered 1 to 8 are in a box. One piece of paper is randomly drawn from the box, its number is recorded, and the paper is returned to the box. Then a 2nd random drawing is made, its number is written down. In how many ways can the sum of the 2 numbers equal 12?

The table below may help.

First Number	4	5	6	7	8
Second Number	8	7	6	5	4

From the table you can see that there are **5 ways** for this to happen.

Ex.2: Eight pieces of paper, numbered 1 to 8 are in a box. Two pieces of paper drawn at the same time from the box, their numbers are recorded. In how many ways can the sum of the 2 numbers equal 12?

The table below may help.

First Number	4	5	7	8
Second Number	8	7	5	4

From the table you can see that there are **4 ways** for this to happen.

The Fundamental Counting Principle:

If event E_1 can occur in m_1 different ways and event E_2 can occur in m_2 different ways, then the number of ways that both E_1 and E_2 can occur is $m_1 m_2$.

Ex3: How many different pairs of letters from the English alphabet are possible?

The 1st letter can be chosen 26 ways. The second letter can be chosen 26 ways.

$26 \times 26 = \mathbf{676}$ different ways

Ex4: Our telephone numbers consist of a 3-digit Area Code and a 7 digit local number. Knowing that the 1st digit of a local number cannot be zero, How many different telephone numbers are possible within the same area code?

The number of choices for each number may be patterned after the values in the table below.

9	10	10	10	10	10	10
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So the number of local telephone numbers is $9(10^6) = 9,000,000$

Occasionally we have to use independent logic as opposed to a predefined formula.

Ex5. In how many ways can a 10-question true-false test be answered if at least 9 questions are answered?

If all questions are answered, then it's $2^{10} = 1024$. If exactly 9 questions are answered, then it's $2^9 \times 10 = 5120$.

So the answer is $1024 + 5120 = \mathbf{6144}$ possible ways.

1. A customer can choose 1 of 4 amplifiers, 1 of 6 compact disc players, and 1 of 5 speaker models for an entertainment system. Determine the number of possible system configurations.
2. An attache case has 2 locks, each of which is a 3 digit number sequence where digits may be repeated. Find a total number of combinations of the 2 locks in order to open the attache a case.
3. How many 3 digit numbers can be formed if the leading digit cannot be zero?
4. How many 3 digit numbers can be formed if all digits are different?
5. in 1963, the united states postal service launched the zoning improvement plan zip code to streamline the mail delivery system. A zip code consists of a 5 digit sequence of numbers. Find the number of zip codes if the first digit is a 1 or a 2.
6. Three couples have reserved seats in a row for a concert. In how many different ways can they be seated if the two members of each couple wish to sit together?
7. In how many ways can five girls and three boys walk through a doorway single file, if the girls walk through before the boys?
8. The nine justices of the U.S. Supreme Court pose for a photograph while standing in a straight line, as opposed to the typical pose of two rows. How many different orders of the justices are possible for this photograph?
9. Eight sprinters have qualified for the finals in the 100-meter dash at the NCAA national track meet. How many different orders of the top three finishers are possible?