

Introduce: Polynomial – Linear, Quadratic, Cubic, etc. Models

Example 1.

Find the general term for the sequence: -1, 4, 13, 26, 43, ...

We set up a set of successive differences and enter them on the line below.

We repeat this procedure until all differences are the same.

The number of new levels gives us the degree of the polynomial for the general term.

a_n	-1	4	13	26	43	
d_1	5	9	13	17		
d_2	4	4	4			

Since the 2nd Differences are the Same, There is a Quadratic Model (2nd Degree).

$$a_n = bn^2 + cn + d$$

We need to Find b, c, and d

We know that $a_1 = -1$. Therefore $b(1)^2 + c(1) + d = -1$

We know that $a_2 = 4$. Therefore $b(2)^2 + c(2) + d = 4$

We know that $a_3 = 13$. Therefore $b(3)^2 + c(3) + d = 13$

This gives 3 equations and 3 unknowns

$$\begin{cases} b + c + d = -1 \\ 4b + 2c + d = 4 \\ 9b + 3c + d = 13 \end{cases} \rightarrow \begin{bmatrix} 1 & 1 & 1 & -1 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 13 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow \begin{cases} b = 2 \\ c = -1 \\ d = -2 \end{cases}$$

Using Augmented Matrices with the aid of a calculator:

The Quadratic Model is: $a_n = 2n^2 - n - 2$

Random Check for a_4 :

$$a_4 = 2(16) - (4) - 2 = 32 - 4 - 2 = 26 \checkmark$$

Example 2.

Find the general term for the sequence: 8, 34, 100, 224, 424, 718, 1124

a_n	8	34	100	224	424	718	1124
d_1	26	66	124	200	294	406	
d_2	40	58	76	94	112		
d_3	18	18	18	18			

Since the 3rd Differences are the same, There is a Cubic Model.

$$a_n = bn^3 + cn^2 + dn + e$$

We need to find b, c, d, and e

$$\begin{cases} b + c + d + e = 8 \\ 8b + 4c + 2d + e = 34 \\ 27b + 9c + 3d + e = 100 \\ 64b + 16c + 4d + e = 224 \end{cases} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 8 \\ 8 & 4 & 2 & 1 & 34 \\ 27 & 9 & 3 & 1 & 100 \\ 64 & 16 & 4 & 1 & 224 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \rightarrow \begin{cases} b = 3 \\ c = 2 \\ d = -1 \\ e = 4 \end{cases}$$

Cubic Model: $a_n = 3n^3 + 2n^2 - n + 4$

Random Check

$$a_5 = 3(125) + 2(25) - 5 + 4 = 375 + 50 - 5 + 4 = 424 \checkmark$$

Find the n th term, then find a_{10} .

1. 0, 3, 10, 21, 36, 55, ...

2. 4, 4, 12, 34, 76, 144, 244, 382,

3. 2, -5, -16, -31, -50, -73, -100, ...

4. -9, -14, -53, -180, -473, -1034, -1989, -3488, -5705, ...