

The function  $f$  is defined by  $f(x) = \sqrt{25 - x^2}$  for  $-5 \leq x \leq 5$ .

a. Find  $f'(x)$

b. Write an equation for the line tangent to the graph of  $f$  at  $x = -3$ .

c. Let  $g$  be the function defined by  $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5 \end{cases}$   
Is  $g$  continuous at  $x = -3$ ? Use the definition of continuity to explain your answer.

d. Find the value of  $\int_0^5 x\sqrt{25 - x^2} dx$

x	1	1.1	1.2	1.3	1.4
f'(x)	8	10	12	13	14.5

The function  $f$  is twice differentiable for  $x > 0$  with  $f(1) = 15$  and  $f''(1) = 20$ . Values of  $f'$ , the derivative of  $f$ , are given for selected values of  $x$  in the table above.

a. Write an equation for the line tangent to the graph of  $f$  at  $x = 1$ . Use this line to approximate  $f(1.4)$ .

b. Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate  $\int_1^{1.4} f'(x) dx$ . Use the approximation for  $\int_1^{1.4} f'(x) dx$  to estimate the value of  $f(1.4)$ . Show the computations that lead to your answer.

c. Use Euler's method, starting at  $x = 1$  with two steps of equal size, to approximate  $f(1.4)$ . Show the computations that lead to your answer.

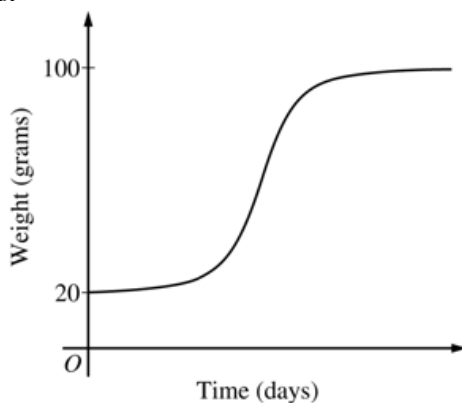
d. Write the second-degree Taylor polynomial for  $f$  about  $x = 1$ . Use the Taylor polynomial to approximate  $f(1.4)$ .

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time  $t = 0$ , when the bird is first weighed, its weight is 20 grams. If  $B(t)$  is the weight of the bird, in grams, at time  $t$  days after it is first weighed, then  $\frac{dB}{dt} = \frac{1}{5}(100 - B)$ .

Let  $y = B(t)$  be the solution to the differential equation above with initial condition  $B(0) = 20$ .

- a. Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

- b. Find  $\frac{d^2B}{dt^2}$  in terms of  $B$ . Use  $\frac{d^2B}{dt^2}$  to explain why the graph of  $B$  cannot resemble the following graph.



- c. Use separation of variables to find  $y = B(t)$ , the particular solution to the differential equation with initial condition  $B(0) = 20$ .

The function  $g$  has derivatives of all orders, and the Maclaurin series for  $g$  is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \dots$$

a. Using the ratio test, determine the interval of convergence of the Maclaurin series for  $g$ .

b. The Maclaurin series for  $g$  evaluated at  $x = \frac{1}{2}$  is an alternating series whose terms decrease in absolute value to 0. The approximation for  $g\left(\frac{1}{2}\right)$  using the first two terms of this series is  $\frac{17}{120}$ . Show that this approximation differs from  $g\left(\frac{1}{2}\right)$  by less than  $\frac{1}{200}$ .

c. Write the first three terms and the general term of the Maclaurin series for  $g'(x)$ .