

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let

$$f(x) = g(x) - \cos\left(\frac{x}{2}\right).$$

a. Find $\int_{-2\pi}^{4\pi} f(x) dx$. Show the computations that lead to your answer.

b. Find all x -values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

c. Let $h(x) = \int_0^{3x} g(t) dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

Let $f(x) = \ln(1 + x^3)$.

- a. The Maclaurin series for $\ln(1 + x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1} \cdot \frac{x^n}{n} + \dots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for f .
- b. The radius of convergence of the Maclaurin series for f is 1. Determine the interval of convergence. Show the work that leads to your answer.
- c. Write the first four nonzero terms of the Maclaurin series for $f'(t^2)$. If $g(x) = \int_0^x f'(t^2) dt$, use the first two nonzero terms of the Maclaurin series for g to approximate $g(1)$.
- d. The Maclaurin series for g , evaluated at $x = 1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from $g(1)$ by less than $\frac{1}{5}$.

t (minutes)	0	4	9	15	20
W(t) (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

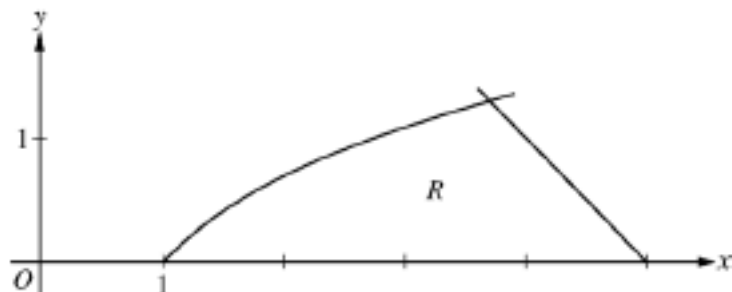
The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- a. Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

- b. Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.

- c. For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

- d. For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?



Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

a. Find the area of R .

b. Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

c. The horizontal line, $y = k$, divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .