Mathematical Induction

Mathematical Induction is frequently used to prove a statement involving n for **all values of n belonging to the set of positive integers**.

In order to prove that a statement is true using Mathematical Induction, the following steps are taken:

1. Establish that the statement is true when n = 1.
2. Assume that the statement is true for some unknown value of n (we will choose n = k)
3. Show what the statement will look like when n = k + 1
4. Using the assumption statement, demonstrate that the statement will also be true for n = k + 1.

Example 1

Prove: $2 + 4 + 6 + 8 + 10 + \ldots + 2n = n(1 + n) \quad \forall n \in N$

1. Show the statement true when n = 1:
   ie: $2 = 1(1 + 1)$ This is true

2. Assume the statement true when n = k
   ie: Assume $2 + 4 + \ldots + 2k = k(1 + k)$

3. Show the statement is also true when n = k + 1
   ie: Show that $2 + 4 + 6 + 8 + \ldots + 2(k + 1) = (k + 1)(1 + k + 1)$

4. Demonstration (Demo)

$2 + 4 + \ldots + 2k = k(1 + k)$	by Assumption
$2 + 4 + 6 + \ldots + 2k + 2(k + 1) = k(1 + k) + 2(k + 1)$	by APE
$= (1 + k)(k + 2)$	Factoring - QED

Example 2

Prove $n^2 - n$ is divisible by 2 $\forall n \in N$

1. Show the statement true for n = 1
   ie: $1^2 - 1$ is divisible by 2 This is true

2. Assume the statement true when n = k
   ie: Assume $k^2 - k$ is divisible by 2

3. Show the statement true for n = k + 1
   ie: $(k + 1)^2 - (k + 1)$ is divisible by 2

4. Demonstration (Demo)

$(k + 1)^2 - (k + 1) = k^2 + 2k + 1 - k - 1$	by Multiplication
$= k^2 + k$	Simplification
$= (k^2 - k) + 2k$	Re-Writing the Expression

The 1st term is divisible by 2 from the assumption
The 2nd term is divisible by 2 by definition
The sum of 2 terms, each divisible by 2 is itself divisible by 2
QED

Pre-Calc 2 Assignment 229 Tuesday, March 31, 2015 Hour Name Ewell-Key

Prove the following True $\forall$ n $\in$ $\mathbb{N}$ using Mathematical Induction.

1. $\sum_{i=1}^{n} i$ = 1 + 2 + 3 + 4 + ... + n = $\frac{n(n+1)}{2}$

2. $\sum_{i=1}^{n} (2i-1)$ = 1 + 3 + 5 + 7 + ... + (2n - 1) = n^2

3. $\sum_{i=1}^{n} i^2$ = $1^2 + 2^2 + 3^2 + 4^2$ + ... + n^2 = $\frac{n(n+1)(2n+1)}{6}$