AP Calculus BC 2 Assignment 229 Tuesday March 31, 2015 Hour Name

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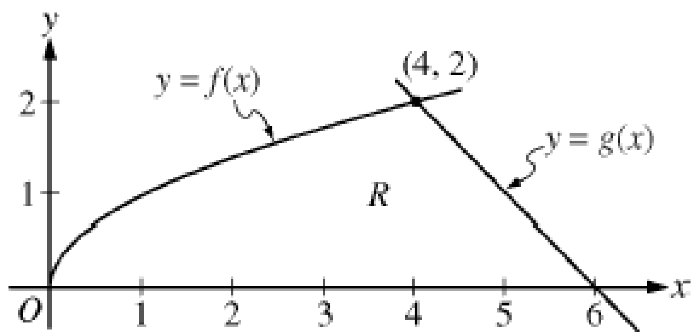


The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

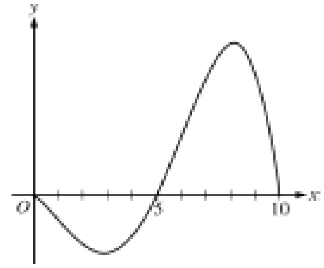
a.	Find the area of R.
b.	The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross-section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is 2y. Write, but do not evaluate, an integral expression that gives the volume of the solid.
c.	There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

a. Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

b. Find all intervals on which the graph of f is concave down. Justify your answer.

c. Given that $f(1) = 2$, determine the function f.



Graph of f

The graph of the differentiable function y = f(x) with domain $0 \le x \le 10$ is shown in the figure above. The area of the region enclosed between the graph of f and the x-axis for $0 \le x \le 5$ is 10, and the area of the region enclosed between the graph of f and the x-axis for $5 \le x \le 10$ is 27. The arc length for the portion of the graph of f between x = 0 and x = 5 is 11, and the arc length for the portion of the graph of f between x = 5 and x = 10 is 18. The function f has exactly two critical points that are located at x = 3 and x = 8.

a. Find the average value of f on the interval $0 \le x \le 5$.

b. Evaluate $\int_0^{10} (3f(x) + 2)\, dx$. Show the computations that lead to your answer.

c. Let $g(x) = \int_5^x f(t)\, dt$. On what intervals, if any, is the graph of g both concave up and descending? Explain your reasoning.

d. The function h is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of h is $h'(x) = f'\left(\frac{x}{2}\right)$. Finds the arc length of the graph of y = h(x) from x = 0 to x = 20.

t (seconds)	0	10	40	60
B(t) (meters)	100	136	9	49
v(t) (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for B(t) and Ben's velocity, v(t), measured in meters per second, at selected times t.

a. Use the data in the table to approximate Ben's acceleration at time t = 5 seconds. Indicate units of measure.

b. Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem.

Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

c. For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

d. A light is directly above the western end of the track. Ben rides so that at time t, the distance L(t) between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time t = 40 ?