AP Calculus BC 2 Assignment 228 Monday March 31, 2015 Hour Name

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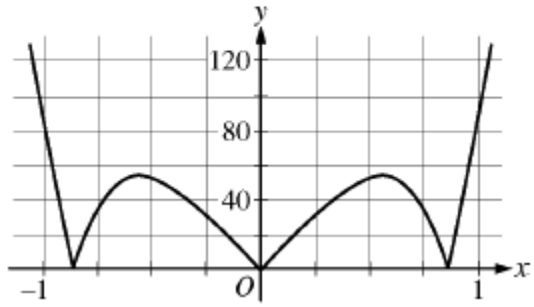

Graph of y = $\left|f^{(5)}(x)\right|$

Let f(x) = sin x^2 + cos x. The graph of y = $|f^{(5)}(x)|$ is shown above.

a.	Write the first four nonzero terms of the Taylor series for sin x about x = 0, and write the first four nonzero terms of the Taylor series for sin x^2 about x = 0.
b.	Write the first four nonzero terms of the Taylor series for cos x about x = 0. Use this series and the series for sin x^2, found in part (a), to write the first four nonzero terms of the Taylor series for f about x = 0.
c.	Find the value of $f^{(6)}(0)$.
d.	Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about x = 0. Using information form the graph of y = $\left|f^{(5)}(x)\right|$ shown above, show that $\left|P_4\left(\frac{1}{4}\right)-f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

A Cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where S(t) is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

a. According to the model, what is the height of the water in the can at the end of the 60-day period?

b. According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

c. Assuming no evaporating occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

d. During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height M(t) is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of the water in the two cans are changing at the same rate.

A 12,000-liter tank of water is filled to capacity. At time t = 0, water begins to drain out of the tank at a rate modeled by r(t), measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

a. Is r continuous at t = 5 ? Show the work that leads to your answer.

b. Find the average rate at which water is draining from the tank between time t = 0 and time t = 8 hours.

c. Find r'(3). Using correct units, explain the meaning of the value in the context of this problem.

d. Write, but do not solve, and equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

The polar curve r is given by $r(\phi) = 3\phi + \sin \phi$, where $0 \le \phi \le 2\pi$.

a. Find the area in the second quadrant enclosed by the coordinate axes and the graph of r.

b. For $\frac{\pi}{2} \le \phi \le \pi$, there is one point P on the polar curve r with x-coordinate -3. Find the angle ϕ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.

c. A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\phi}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\phi = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.