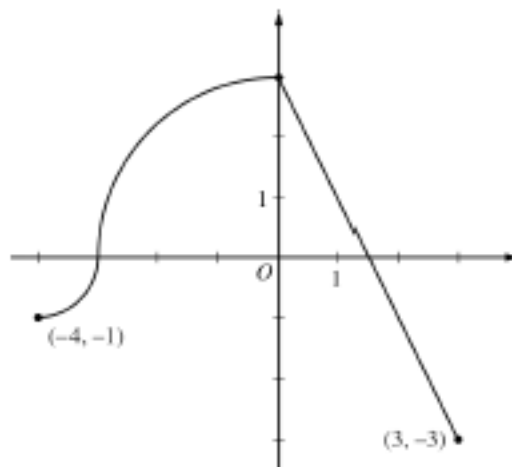


Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f , the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.

- a. Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k .

- b. The region R is rotated about the x -axis to form a solid. Find the volume, V , of the solid in terms of k .

- c. The volume V , found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

Graph of f

The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t) dt$.

- | |
|---|
| a. Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$. |
| b. Determine the x -coordinate of the graph at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. |
| c. Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer. |
| d. Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem. |

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

a. Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time = $\frac{1}{4}$).

b. Find $\frac{d^2W}{dt^2}$ in terms of W . Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

c. Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

Let f be a function defined by $f(x) = \begin{cases} 1 - 2 \sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0 \end{cases}$

a. Show that f is continuous at $x = 0$

b. For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

c. Find the average value of f on the interval $[-1, 1]$.