

A geometric sequence has a constant ratio, r , between terms.

Example: 3, 6, 12, 24, 48, 96, ... is a Geometric Sequence with $r = 2$.

In general, any geometric sequence has a general term: $a_n = a_1(r - 1)^n$

This gives the n^{th} term of a geometric sequence when the 1^{st} term is a_1 .

The formula for the sum of the 1^{st} n terms of a geometric series is: $\frac{a_1(r^n - 1)}{r - 1}$

Geometric Series Sum of an infinite # of terms when $-1 < r < 1$: $\frac{a_1}{1 - r}$

Sigma Notation

$\sum_{i=1}^n 3i$ is an example of an Arithmetic Series: $3 + 6 + 9 + 12 + \dots + (3n)$ has a constant difference of 3

$\sum_{i=1}^n 10(2^{i-1})$ is an example of a Geometric Series: $10 + 20 + 40 + 80 + \dots + 10(2^{n-1})$ has a constant ratio of 2

Formula for the sum of a Finite Geometric Series: $\frac{a_1(1 - r^n)}{1 - r}$

$$\sum_{n=1}^8 6(2^{n-1}) = 6 + 12 + 24 + 48 + 96 + 192 + 384 + 768 = \frac{6(1 - 2^8)}{1 - 2} = 1530$$

Formula for the sum of an Infinite Geometric Series where $|r| < 1$: $\frac{a_1}{1 - r}$

$$\sum_{n=1}^{\infty} 243\left(\frac{1}{3}\right)^{n-1} = 243 + 81 + 27 + 9 + \dots = \frac{243}{1 - 1/3} = \frac{729}{2}$$

$7 + 9 + 11 + 13 + 15 + \dots + 571$ is a finite arithmetic series

Generator: $7 + (n - 1)2$ or $2n + 5$

Number of Terms: $2n + 5 = 571 \square 2n = 566 \square n = 283$

Sigma Notation: $\sum_{i=1}^{283} 2n + 5$

$$\text{Sum: } \frac{283(7 + 571)}{2} = \frac{283(578)}{2} = \frac{163574}{2} = 81787$$

Example of a sequence of partial sums:

Given a Series such as: $1 + 2 + 3 + 4 + 5 + 6 + \dots$

The sequence of partial sums is: 1, 3, 6, 10, 15, 21,

1. Write the 1st five terms of $a_n = 4n - 7$

2. Write the 1st five terms of $a_n = \frac{2n}{n+1}$

3. Write the 1st five terms and the general term of the geometric sequence, where $a_1 = 6$ and $r = 3$.

4. Write the 1st five terms and the general term of the geometric sequence, where $a_1 = 4$ and $r = 2$.

5. Find the n^{th} term of the geometric sequence where $a_1 = 5$, $a_3 = \frac{45}{4}$, $n = 8$

6. 3, 36, 432, ... Write in Sigma form, then find the 7th term.

7. Find the 1st four terms of the sequence of partial sums for the sequence: 8, -4, 2, -1, $\frac{1}{2}$, ...

8.
$$\sum_{n=1}^{\infty} 16 \left(\frac{1}{2} \right)^{n-1} =$$

9.
$$\sum_{n=1}^9 2^{n-1} =$$

10.
$$\sum_{n=1}^9 (-2)^{n-1} =$$