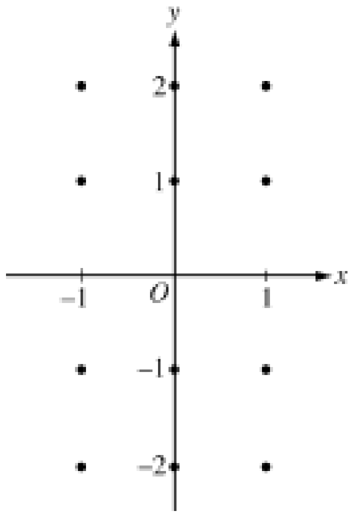
Consider the differential equation $\frac{dy}{dx} = \frac{x + 1}{y}$

a. On the axes provided, sketch a slope field for the given differential equation at the twelve-points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point (0, -1).



b. While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$

Let f and g be the functions defined by $f(x) = \frac{1}{x}$ and $g(x) = \frac{4x}{1 + 4x^2}$, for all $x > 0$.

a. Find the absolute maximum value of g on the open interval $(0, \infty)$ if the maximum exits. Find the absolute minimum value of g on the open interval $(0, \infty)$ if the minimum exits. Justify you answers.

b. Find the area of the unbounded region in the first quadrant to the right of the vertical line $x = 1$, below the graph of f, and above the graph of g.

Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

a. For $0 \le t \le 6$, find all times t during which particle R is moving to the right.
b. For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.
c. Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.
d. Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

The Maclaurin series for the function f is given by $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$ on its interval of convergence.

a. Find the interval of convergence for the Maclaurin series of f. Justify your answer.

b. Show that $y = f(x)$ is a solution to the differential equation $xy' - y = \frac{4x^2}{1 + 2x}$ for $|x| < R$, where R is the radius of convergence from part (a).