

An **Infinite Sequence** is a **function** whose **domain** is the set of **positive integers**:  $\{1, 2, 3, 4, 5, 6, \dots\}$   
The function values  $a_1, a_2, a_3, a_4, \dots, a_n, \dots$  are the **terms** of the sequence.

If the domain of a function consists of the first  $n$  positive integers only, the sequence is a **finite sequence**.

Occasionally it may be convenient to allow the domain to be **whole numbers**:  $\{0, 1, 2, 3, 4, \dots\}$   
In this case the values are regarded as  $a_0, a_1, a_2, a_3, a_4, \dots$

Example: Write the 1<sup>st</sup> 4 terms of the following sequence:  $a_n = 3n - 2$

Solution:

$$a_1 = 3(1) - 2 = 1$$

$$a_2 = 3(2) - 2 = 4$$

$$a_3 = 3(3) - 2 = 7$$

$$a_4 = 3(4) - 2 = 10$$

The sequence contains elements:  $1, 4, 7, 10, \dots, (3n - 2), \dots$

Example: Write the 1<sup>st</sup> 5 terms of:  $a_n = \frac{(-1)^n}{2n-1}$

Solution:  $-1, \frac{1}{3}, -\frac{1}{5}, \frac{1}{7}, -\frac{1}{9}, \dots, \frac{(-1)^n}{2n-1}, \dots$

Find the apparent  $n^{\text{th}}$  term of  $3, 5, 7, 9, 11, \dots$

Solution:  $a_n = 2n - 1$

Sometimes the  $n^{\text{th}}$  term is referred to as a **generator**.

Sigma notation, indicated by  $\sum_{i=1}^n a_i$ , may be used to indicate the sum of the terms in a sequence.

When we **add the terms of a sequence**, we build a **series**.

An **Arithmetic Sequence** has a constant difference,  $d$ , between terms.

Example:  $5, 8, 11, 14, 17, 20, \dots$  is an Arithmetic Sequence with  $d = 3$ .

The generator is  $a_n = 3n + 2$

A **Geometric Sequence** has a constant ratio,  $r$ , between terms. That is if we divide any term by its predecessor, we get the same ratio,  $r$ , each time.

Example:  $3, 6, 12, 24, 48, 96, \dots$  is a Geometric Sequence with  $r = 2$ .

A sequence can be defined **Recursively** when each successive term is defined in terms of earlier terms. In this case, the 1<sup>st</sup> term has to be given.

Example:  $a_1 = 7, a_{n+1} = 2a_n - 3$  will give  $7, 11, 19, 35, 67, \dots$

1.  $a_n = 4n - 7$ . Write the 1st five terms of the sequence, where n starts at 1

2.  $\sum_{n=1}^5 4n - 7 =$

3.  $a_n = \frac{2n}{n+1}$ . Write the 1st five terms of the sequence, where n starts at 1

4.  $\sum_{n=1}^5 \frac{2n}{n+1} =$

5.  $a_1 = 6, a_{n+1} = 2a_n$  Find  $a_6$ .

6.  $\sum_{n=1}^x a_n = 3 + 5 + 7 + 9 + \dots + 21$ . Find the expression for  $a_n$ .

7.  $\sum_{n=1}^{\infty} a_n = 2 + 5 + 8 + 11 + 14 + 17 + 20 + \dots$  Find the expression for  $a_n$ .

8. Using the information from #7, write the 219<sup>th</sup> Term.