AP Calculus BC Assignment 223 Friday, March 20, 2015 Hour Name

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Mike's driveway is clear of snow at midnight. From midnight to 9:00 AM, accumulation of snow on the driveway is at a rate of $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is the number of hours after midnight. Snow removal starts at 6 AM (t = 6) at the rate of $g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9 \end{cases}$ cubic feet per hour.

a. How many cubic feet of snow have accumulated on the driveway by 6 AM?

b. Find the rate of change of the volume of snow on the driveway at 8 AM.

c. Let h(r) represent the total amount of snow, in cubic feet, that has been removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

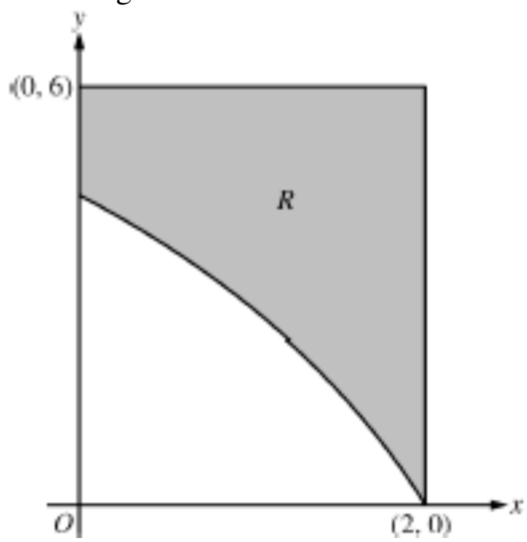
d. How many cubic feet of snow are on the driveway at 9 AM?

$$f(x) = \begin{cases} \dfrac{\cos x - 1}{x^2} & \text{for } x \neq 0 \\ -\dfrac{1}{2} & \text{for } x = 0 \end{cases}$$

The function f, defined above, has derivatives of all orders. Let g be the function defined by

$g(x) = 1 + \int_0^x f(t)\,dt$.

a.	Write the first three nonzero terms and the general term of the Taylor series for cos x about x = 0. Use this series to write the **first three nonzero** terms and the **general term** of the Taylor series for f about x = 0.
b.	Use the Taylor series for f about x = 0 found in part (a) to determine whether f has a relative maximum, relative minimum, or neither at x = 0. Give a reason for your answer.
c.	Write the fifth-degree Taylor polynomial for g about x = 0.
d.	The Taylor series for g about x = 0, evaluated at x = 1, is an alternating series with individual terms that decrease in absolute value to 0. Use the third-degree Taylor polynomial for g about x = 0 to estimate the value of g(1). Explain why this estimate differs from the actual value of g(1) by less than $\frac{1}{6!}$.



In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4 \ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

a.	Find the area of R.
b.	Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.
c.	The region R is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is a square. Find the volume of the solid.

The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right) \cos\left(x + \frac{1}{x}\right)$.

a. Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

b. On what subintervals of (0.12, 1), if any, is the graph of g concave down? Justify your answer.

c. Write an equation for the line tangent to the graph of g at $x = 0.3$.

d. Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?