

Find the Multiplicative Inverse of a 3x3 Matrix,

Using Multiple Augmented Matrices

Gaussian Elimination requires zeros below the major diagonal

Gauss-Jordan goes further: zeros above as well as below the major diagonal

Original Matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Augmented the Matrix with additional columns that represent the Identity Matrix.

Take steps to make the 1st three columns become the Identity. The remaining 3 columns will be the Inverse.

$$\begin{bmatrix} 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 2 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1/2 & 1/2 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}$$

Inverse:

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & -1/2 & 1/2 \\ 1 & 0 & -1 \end{bmatrix}$$

Find the Multiplicative Inverse of this 4x4 Matrix

$$\begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -2 & -2 & 1 & 0 & 0 & 0 \\ 3 & -5 & -2 & -3 & 0 & 1 & 0 & 0 \\ 2 & -5 & -2 & -5 & 0 & 0 & 1 & 0 \\ -1 & 4 & 4 & 11 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & -1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -2 & 0 & 1 & 0 \\ 0 & 2 & 3 & 9 & 1 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 4 & -5 & 2 & 0 & 0 \\ 0 & 1 & 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -5 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 7 & -2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & -5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -24 & 7 & 1 & -2 \\ 0 & 1 & 0 & 0 & -10 & 3 & 0 & -1 \\ 0 & 0 & 1 & 0 & -29 & 7 & 3 & -2 \\ 0 & 0 & 0 & 1 & 12 & -3 & -1 & 1 \end{bmatrix}$$

Inverse:

$$\begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix}$$

1.
$$\begin{bmatrix} -1/2 & 3/4 & 1/4 \\ 1 & 0 & -3/2 \\ 0 & -1 & 1/2 \end{bmatrix}$$

2.
$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$