

## Finding the Multiplicative Inverse of a 3x3 Matrix:

Original Matrix

$$\begin{bmatrix} 5 & 7 & 2 \\ 1 & -1 & 0 \\ 3 & -2 & 4 \end{bmatrix}$$

Find the Matrix of Minors

$$\begin{bmatrix} -4 & 4 & 1 \\ 32 & 14 & -31 \\ 2 & -2 & -12 \end{bmatrix}$$

The Matrix of Co-Factors

$$\begin{bmatrix} -4 & -4 & 1 \\ -32 & 14 & 31 \\ 2 & 2 & -12 \end{bmatrix}$$

Determinant of Original

$$5 \begin{vmatrix} -1 & 0 \\ -2 & 4 \end{vmatrix} - 7 \begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} + 2 \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -46$$

Transpose

$$\begin{bmatrix} -4 & -32 & 2 \\ -4 & 14 & 2 \\ 1 & 31 & -12 \end{bmatrix}$$

Inverse: Divide Transpose by Determinant

$$\begin{bmatrix} 2/23 & 16/23 & -1/23 \\ 2/23 & -7/23 & -1/23 \\ -1/46 & -31/46 & 6/23 \end{bmatrix}$$

Check

$$\begin{bmatrix} 5 & 7 & 2 \\ 1 & -1 & 0 \\ 3 & -2 & 4 \end{bmatrix} * \begin{bmatrix} 2/23 & 16/23 & -1/23 \\ 2/23 & -7/23 & -1/23 \\ -1/46 & -31/46 & 6/23 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Original Matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Find the Matrix of Minors

$$\begin{bmatrix} -2 & 0 & 2 \\ -2 & -1 & 0 \\ 2 & -1 & -2 \end{bmatrix}$$

The Matrix of Co-Factors

$$\begin{bmatrix} -2 & 0 & 2 \\ 2 & -1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$$

Determinant of Original

$$-2 + 0 + 4 = 2$$

Transpose

$$\begin{bmatrix} -2 & 2 & 2 \\ 0 & -1 & 1 \\ 2 & 0 & -2 \end{bmatrix}$$

Inverse: Divide Transpose by Determinant

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & -1/2 & 1/2 \\ 1 & 0 & -1 \end{bmatrix}$$

Check

$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1/2 & 1/2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the Inverse of the 3 x 3 matrix via Matrix of Minors, Matrix of Co-Factors, Determinant, Transpose

1. 
$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$$

2. 
$$\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$$

3. 
$$\begin{bmatrix} -5 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & 5 & 7 \end{bmatrix}$$

4. 
$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 5 & 0 \\ 2 & 5 & 0 \end{bmatrix}$$