

Special (2nd) method of calculating 3 x 3 Determinant:

$$\text{Find } \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

1. Augment the matrix with the 1st 2 columns $\begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix}$
2. Multiply along downward diagonals starting with a and add all products.
3. Multiply along upward diagonals starting with g and add all products.
4. Subtract the result from #3 from the result from #2.
5. Answer: $[(aei + bfg + cdh)] - [(gec) + (hfa) + (idb)]$

Review methods of finding Triangular Area:

1. $\frac{1}{2} ab$, Given altitude and base
2. Heron's Formula, given 3 sides
3. $\frac{1}{2} ab \sin C$, given 2 sides and an included angle

Introduce the use of 3 by 3 determinants:

A(d, e), B(f, g), C(h, i)

$$\text{Area is } \frac{1}{2} \begin{vmatrix} d & e & 1 \\ f & g & 1 \\ h & i & 1 \end{vmatrix}$$

When 3 points are collinear, the area of a constructed triangle = 0
The above determinant may be used to test collinearity of 3 points.

Cramer's Rule for solving 3 equations and 3 unknowns.

$$\begin{cases} 8x - 7y + 10z = 15 \\ 2x + 3y + 8z = 7 \\ -4x + 5y - 2z = -9 \end{cases}$$

$$x = \frac{\begin{vmatrix} 15 & -7 & 10 \\ 7 & 3 & 8 \\ -9 & 5 & -2 \end{vmatrix}}{\begin{vmatrix} 8 & -7 & 10 \\ 2 & 3 & 8 \\ -4 & 5 & -2 \end{vmatrix}} = \frac{336}{48} = 7$$

(7, 3, -2)

Show Work

1. Using Matrices, Find the Area of $\triangle ABC$, with $A(-9, 12)$, $B(2, 7)$, $C(50, 21)$

2. Given $A(21, 9)$, $B(56, 19)$, $C(20, x)$ are colinear points. Use Matrices to find x .

3. Solve, using Cramer's Rule:
$$\begin{cases} x + 2y - 2z = -13 \\ 3x + 4y + 2z = -1 \\ x + 2y + z = -1 \end{cases}$$