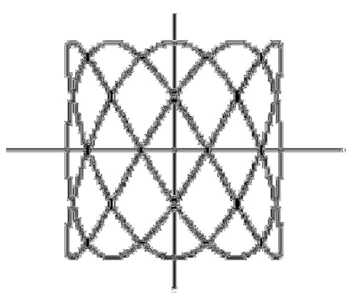
Vector-Valued Functions:

Suppose a particle moves along a smooth curve in the plane so that its position at any time is t is **r**(t) = <x(t), y(t)> where x and y are differentiable functions of t.

1. The particle's **position vector** is **r**(t) = <x(t), y(t)>.
2. The particle's **velocity vector** is **v**(t) = $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$
3. The particle's **speed** is the magnitude of **v**, denoted $|\mathbf{v}|$. Speed is a scalar, not a vector.
   $|\mathbf{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$
4. The particle's **acceleration vector** is **a**(t) = $\left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$
5. The particle's **direction vector** is the **direction vector** $\frac{\mathbf{v}}{|\mathbf{v}|}$.

6. A particle moves in the plane with a position vector **r**(t) = <sin 3t, cos 5t>.
   a. Find the velocity vector
   b. Findd the acceleration vector
   c. Using a calculator, determine the path of the particle.

Solution:
   a. Velocity v(t) = <3 cos 3t, -5 sin 5t>.
   b. Acceleration a(t) = <-9 sin 3t, -25 cos 5t>.
   c. The path of the particle is found by graphing in parametric mode, the curve defined by x = sin 3t and y = cos 5t.

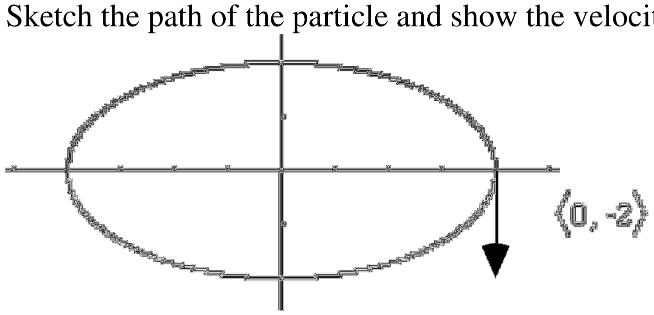


7. A particle moves in an elliptical path so that its position at any time $t \geq 0$ is given by $\mathbf{r}(t) = \langle 4 \sin t, 2 \cos t \rangle$.

a. Find the velocity and acceleration vectors.
$\mathbf{v}(t) = \langle 4 \cos t, -2 \sin t \rangle$
$\mathbf{a}(t) = \langle -4 \sin t, -2 \cos t \rangle$

b. Find the velocity, acceleration, speed, and direction of motion at $t = \frac{\pi}{4}$.
$\mathbf{v}\left(\frac{\pi}{4}\right) = 2\sqrt{2}, -\sqrt{2}$
$\mathbf{a}\left(\frac{\pi}{4}\right) = -2\sqrt{2}, -\sqrt{2}$
Speed = $\left|\mathbf{v}\left(\frac{\pi}{4}\right)\right| = \sqrt{8 + 2} = \sqrt{10}$
Slope = $\tan \theta = -\frac{1}{2}$ ∅ $\theta = -0.463648 = -26.5651^\circ$

c. Sketch the path of the particle and show the velocity vector at the point (4, 0)



d. Does the particle travel clockwise or counterclockwise around the origin?
The particle travels clockwise.

Displacement and Distance Traveled

When a particle moves along a line with velocity v(t), the displacement (or net distance traveled) from time t = a to time t = b is given by $\int_a^b v(t)\,dt$, while the (total) distance traveled is given by $\int_a^b |v(t)|\,dt$. When a particle moves in the plane with velocity vector **v**(t), displacement and distance traveled can be found by applying the same integrals to the vector **v**, although in slightly different ways.

Suppose a particle moves along a path in the plane so that its velocity at any time t is **v**(t) = < v_1(t), v_2(t) >, where v_1 and v_2 are integrable functions of t.

The **displacement** from t = a to t = b is given by $\left\langle \int_a^b v_1(t)\,dt, \int_a^b v_2(t)\,dt \right\rangle$.

This vector is added to the position at time t = a to get the position at time t = b.

The **distance traveled** from t = a to t = b is $\int_a^b |v(t)|\,dt = \int_a^b \sqrt{(v_1(t))^2 + (v_2(t))^2}\,dt$.

AP Calculus BC 2 Assignment 218 Friday, March 13, 2015 Hour Name ______________________________

For #'s 1-5: Find $\mathbf{r}'(t)$

1. $\mathbf{r}(t) = 6t\mathbf{i} - 7t^2\mathbf{j} + t^3\mathbf{k}$

2. $\mathbf{r}(t) = \frac{1}{t}\ \mathbf{i} + 16t\mathbf{j} + \frac{t^2}{2}\ \mathbf{k}$

3. $\mathbf{r}(t) = a \cos^3 t\mathbf{i} + a \sin^3 t\mathbf{j} + \mathbf{k}$

4. $\mathbf{r}(t) = 4\sqrt{t}\ \mathbf{i} + t^2\sqrt{t}\ \mathbf{j} + \ln t^2\mathbf{k}$

5. $\mathbf{r}(t) = e^{-t}\mathbf{i} + 4\mathbf{j}$

6. $\int (2t\mathbf{i} + \mathbf{j} + \mathbf{k})\, dt =$

7. $\int \left(\frac{1}{t}\mathbf{i} + \mathbf{j} - t^{3/2}\,\mathbf{k} \right) dt =$

8. $\int \left((2t - 1)\mathbf{i} + 4t^3\mathbf{j} + 3\sqrt{t}\ \mathbf{k} \right) dt =$

A particle moves in a plane with velocity vector $\mathbf{v}(t) = < t - 3\pi \cos \pi t, 2t - \pi \sin \pi t >$. At $t = 0$, the particle is at the point (1, 5).

9. Find the position of the particle at $t = 4$.

10. What is the total distance traveled by the particle from $t = 0$ to $t = 4$?

11. Determine the path that the particle travels going from (1, 5) to (9, 21)