Parametric Area:
Let x = f(t), y = g(t), and the parametric curve exist above the x-axis from t = c to t = d where c < d.

The area between the x-axis and the polar curve is: $\int_c^d g(t)\, f'(t)\, dt$

Parametric Arc Length:
Let x = f(t), y = g(t), represent a parametric curve from t = c to t = d where c < d.

The arc length is: $\int_c^d \sqrt{(f'(t))^2 + (g'(t))^2}\ dt$

Polar Area:
Let r = f(θ) represent a function in polar form.
Recall from Pre-Calculus the relationships between x, y, r, and θ.

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$

$\theta = \arctan\frac{y}{x}$

Polar Area is: $\frac{1}{2}\int_a^b r^2\, d\theta$

Polar Arc Length:

Parametric: Since $\begin{cases} x = r\cos ø \\ y = r\sin ø \end{cases}$ and $\int_a^b \sqrt{\left(\frac{dx}{dø}\right)^2 + \left(\frac{dy}{dø}\right)^2}\, dø$ Also $\begin{cases} \frac{dx}{dø} = -r\sin ø + \cos ø\,\frac{dr}{dø} \\ \frac{dy}{dø} = r\cos ø + \sin ø\,\frac{dr}{dø} \end{cases}$

$\left(\frac{dx}{dø}\right)^2 = r^2\sin^2ø - 2r\sin ø\cos ø\,\frac{dr}{dø} + \cos^2ø\left(\frac{dr}{dø}\right)^2$ and $\left(\frac{dy}{dø}\right)^2 = r^2\cos^2ø + 2r\sin ø\cos ø\,\frac{dr}{dø} + \sin^2ø\left(\frac{dr}{dø}\right)^2$

$$\left(\frac{dx}{dø}\right)^2 + \left(\frac{dy}{dø}\right)^2 = r^2\sin^2ø - 2r\sin ø\cos ø\,\frac{dr}{dø} + \cos^2ø\left(\frac{dr}{dø}\right)^2 + r^2\cos^2ø + 2r\sin ø\cos ø\,\frac{dr}{dø} + \sin^2ø\left(\frac{dr}{dø}\right)^2$$

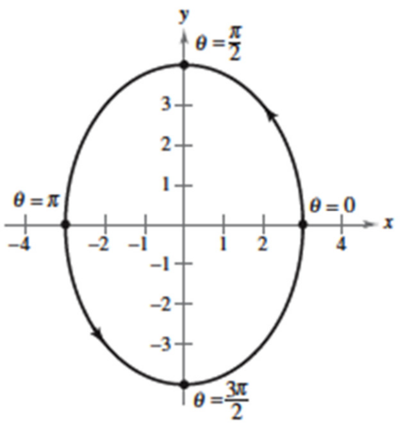
$$\left(\frac{dx}{dø}\right)^2 + \left(\frac{dy}{dø}\right)^2 = r^2(\sin^2ø + \cos^2ø) + \left(\frac{dr}{dø}\right)^2(\sin^2ø + \cos^2ø)$$

$$\left(\frac{dx}{dø}\right)^2 + \left(\frac{dy}{dø}\right)^2 = r^2 + \left(\frac{dr}{dø}\right)^2$$

We have successfully eliminated x and y.

Polar: $\int_a^b \sqrt{r^2 + \left(\frac{dr}{dø}\right)^2}\, dø$

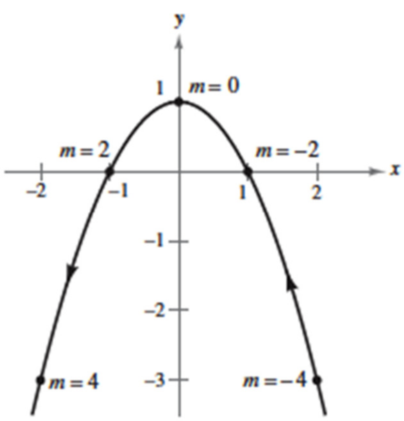
AP Calculus BC 2 Assignment 216 Wednesday, March 11, 2015 Hour Name



$x = 3\cos\theta,\ \ y = 4\sin\theta\ ,\ 0 \le \theta \le 2\pi$

1. Find the arc length defined above.

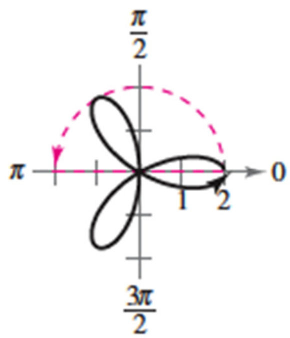
2. Find the area enclosed above.



$x = -\frac{1}{2}m \quad y = 1 - \frac{m^2}{4}$

3. Find the arc length defined above on $-4 \le m \le 4$

4. Find the area enclosed defined above on $-4 \le m \le 4$



r = 2 cos θ

5. Find the area enclosed