

Systems of Linear Equations:

1. Substitution: Isolate a Variable in one equation. Apply that Isolation to the other equation. Solve for the remaining variable. Use that variable to solve for the Isolated Variable.

Example 1

$$\begin{cases} 3x + 2y = -4 \\ 2x + y = -1 \end{cases}$$

$$y = -2x - 1$$

Isolation

$$3x + 2(-2x - 1) = -4$$

Apply Isolation to the Other Equation

$$3x - 4x - 2 = -4$$

$$-x = -2$$

$$x = 2$$

Solve for x

$$y = -2(2) - 1$$

Substitute into the Isolation

$$y = -4 - 1$$

$$y = -5$$

Solve for the Isolated Variable

$$(2, -5)$$

Solution

2. Linear Combination: Multiply one or both equations to make the variable terms opposite. Combine the equations and solve the new equation. At this point, you may start over and eliminate the other variable and repeat, or use the original solution to find the second variable.

Example 2

$$\begin{cases} 2x - 3y = -15 \\ 3x + 2y = 23 \end{cases}$$

$$4x - 6y = -30$$

Multiply Top Equation by 2

$$9x + 6y = 69$$

Multiply 2nd Equation by 3

$$\hline 13x = 39$$

Add the Equations Together

$$x = 3$$

Solve for x

$$6x - 9y = -45$$

Multiply Top Equation by 3

$$\hline -6x - 4y = -46$$

Multiply 2nd Equation by -2

$$\hline -13y = -91$$

Add the Equations Together

$$y = 7$$

Solve for y

$$(3, 7)$$

Solution

3. Evaluating a 2 x 2 Determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. Multiply along the Major Diagonal, then subtract the product along the secondary diagonal.

Example 3

$$\begin{vmatrix} 2 & -3 \\ -5 & 8 \end{vmatrix} = (2)(8) - (-5)(-3) = 16 - 15 = 1$$

4. Cramer's Rule: Rearrange each equation in the form $ax + by = c$. To find x , make a fraction of determinants where the numerator has the constant in place of the x -coefficient, and the denominator using the x and y coefficients. To find y , make a fraction of determinants where the numerator has the constant in place of the y -coefficient, and the denominator using the x and y coefficients.

Example 4

$$\begin{cases} 3x + 2y = 6 \\ 5x + 8y = -4 \end{cases}$$

$$x = \frac{\begin{vmatrix} 6 & 2 \\ -4 & 8 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 5 & 8 \end{vmatrix}} = \frac{(6)(8) - (-4)(2)}{(3)(8) - (5)(2)} = \frac{48 + 8}{24 - 10} = \frac{56}{14} = 4$$

$$y = \frac{\begin{vmatrix} 3 & 6 \\ 5 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 5 & 8 \end{vmatrix}} = \frac{(3)(-4) - (5)(6)}{(3)(8) - (5)(2)} = \frac{-12 - 30}{24 - 10} = \frac{-42}{14} = -3$$

(4, -3)

Solution

Exer. 1-: Solve, using Substitution.

$$1. \begin{cases} 3x + y = 1 \\ 5x - 2y = 20 \end{cases} \quad (2, -5)$$

$$2. \begin{cases} x - y = 0 \\ 5x - 3y = 10 \end{cases} \quad (4, -\frac{1}{2})$$

Solve the Following Systems of Linear Equations

Exer. 1-2: Use Substitution. Exer. 3-5: Start With Linear Combination. Exer. 6-8: Use Cramer's Rule.

1.
$$\begin{cases} x - y = 0 \\ 5x - 3y = 10 \end{cases}$$

2.
$$\begin{cases} 3x + y = 1 \\ 5x - 2y = 20 \end{cases}$$

3.
$$\begin{cases} x + 2y = 1 \\ 5x - 4y = -23 \end{cases}$$

4.
$$\begin{cases} 2x - y = -2 \\ 4x + y = 5 \end{cases}$$

5.
$$\begin{cases} -x + 2y = 2 \\ 3x + y = 15 \end{cases}$$

6.
$$\begin{cases} -x + 2y = 1 \\ x - y = 2 \end{cases}$$

7.
$$\begin{cases} 7x + 8y = 24 \\ x - 8y = 8 \end{cases}$$

8.
$$\begin{cases} x - 3y = -2 \\ 5x + 3y = 17 \end{cases}$$