Parametric Equations

x = f(t) and y = g(t)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}\text{, where } \frac{dx}{dt} \neq 0$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}$$

x = sin t, y = cos t

$$\frac{dy}{dx} = \frac{\text{-sin t}}{\text{cos t}}$$

Example

$x = \sqrt{t}$ and $y = \frac{1}{4}(t^2 - 4)$, $t \geq 0$. Find the slope and concavity at the point (2, 3)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(1/2)t}{(1/2)t^{-1/2}} = t^{3/2}$$

At the point (2, 3), $2 = \sqrt{t} \rightarrow t = 4$ or $3 = \frac{1}{4}(t^2 - 4) \rightarrow 12 = \left(t^2 - 4\right) \rightarrow t^2 = 16 \rightarrow t = 4$.

The Slope is $4^{3/2} = 8$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[dy/dx\right]}{dx/dt} = \frac{\frac{d}{dt}\left[t^{3/2}\right]}{dx/dt} = \frac{(3/2)t^{1/2}}{(1/2)t^{-1/2}} = 3t$$

When t = 4, the 2nd Derivative is 12 > 0, therefore the graph is concave up.

Parametric Form for Arc Length:

Let C be a smooth curve C, with x = f(t), y = g(t), C does not intersect itself on the interval a < t < b, except possibly at the endpoints, then the arc length of C on the interval is:

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$$

AP Calculus BC 2 Assignment 214, Monday, March 2, 2015 Hour Name ______________________

1. $x = t^2$, $y = 5 - 4t$ Find $\frac{dy}{dx}$.

2. $x = \sqrt[3]{t}$, $y = 4 - t$ Find $\frac{dy}{dx}$.

3. $x = \sin^2\theta$, $y = \cos^2\theta$ Find $\frac{dy}{dx}$.

4. $x = 2e^{\theta}$, $y = e^{\theta/2}$ Find $\frac{dy}{dx}$.

5. $x = 2t$, $y = 3t - 1$ At $t = 3$, Find $\frac{dy}{dx}$.

6. $x = \sqrt{t}$, $y = 3t - 1$ At $t = 3$, Find $\frac{d^2y}{dx^2}$.

7. $x = t + 1$, $y = t^2 + 3t$ At $t = -1$, Find $\frac{d^2y}{dx^2}$.

8. $x = t^2 + 3t + 2$, $y = 2t$ At $t = 0$, Find $\frac{d^2y}{dx^2}$.

9. $x = 2\cos\theta$, $y = 2\sin\theta$ At $t = \frac{\pi}{4}$, Find $\frac{d^2y}{dx^2}$.

10. $x = 2 + \cos\theta$, $y = 3\sin\theta$ At $\theta = 0$, Find $\frac{d^2y}{dx^2}$.