Geometric Power Series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}\ ,\ |r|<1$$

Let a = 1 and r = x $\rightarrow$ $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots,\ |x|<1$

This series represents $f(x) = \frac{1}{1-x}$ only on the interval (-1, 1), where f is defined on all $x \neq 1$.

To represent another interval, we must develop a different series with another center.

Construct a Series Centered at -1:

$$\frac{1}{1-x} = \frac{1}{2-(x+1)} = \frac{1/2}{1-\frac{x+1}{2}} = \frac{a}{1-r} \rightarrow a = \frac{1}{2} \text{ and } r = \frac{x+1}{2} \rightarrow |x+1|<2$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{x+1}{2}\right)^n = \frac{1}{2}\left[1+\frac{(x+1)}{2}+\frac{(1+x)^2}{4}+\frac{(1+x)^3}{8}+\ldots.\right],\ |x+1|<2 \rightarrow \text{Convergent on (-3, 1).}$$

Find a power series for f(x) = $\frac{4}{x+2}$ centered at 0.

$$\frac{4}{2+x} = \frac{2}{1-\left(-x/2\right)} = \frac{a}{1-r} \rightarrow a = 2,\ r = -\frac{x}{2} \rightarrow \frac{4}{x+2} = \sum_{n=0}^{\infty} 2\left(-\frac{x}{2}\right)^n = 2\left(1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^8}{8}+\ldots\right)$$

This series is convergent when $\left|-\frac{x}{2}\right|<1 \rightarrow |x|<2 \rightarrow$ Radius = 2 $\rightarrow$ Interval of Convergence is (-2, 2).

Find a power series for f(x) = $\frac{3x-1}{x^2-1}$. Using Partial Fraction Decomposition: f(x) = $\frac{2}{x+1}+\frac{1}{x-1}$

$$\frac{2}{1-(-x)}+\frac{-1}{1-x} = \sum_{n=0}^{\infty} 2\,(-1)^n\ x^n + \sum_{n=0}^{\infty}(-1)x^n = \sum_{n=0}^{\infty}\left[2(-1)^n-1\right]x^n = 1 - 3x + x^2 - 3x^3 + x^4 - \ldots$$

The Interval of Convergence is (-1, 1)

AP Calculus BC 2 Assignment 212 Tuesday, February 24, 2014 Hour Name

1. $f(x) = \frac{1}{2+x}$ Find a geometric power series centered at 0. Then find the interval of convergence.

2. $f(x) = \frac{1}{2+x}$ Find a geometric power series centered at 5. Then find the interval of convergence.

3. $f(x) = \frac{3}{2x-1}$ Find a geometric power series centered at 3. Then find the interval of convergence.

4. $f(x) = \frac{1}{2x-5}$ Find a geometric power series centered at -3. Then find the interval of convergence.

5. $f(x) = \frac{3x}{x^2+x-2}$ Find a geometric power series centered at 0. Then find the interval of convergence.

6. $f(x) = \frac{2}{1-x^2}$ Find a geometric power series centered at 0. Then find the interval of convergence.