

$P_n(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x - c)^n$  is called the **nth Taylor Polynomial** for  $f$  at  $c$ .

If  $c = 0$ ,

then  $P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n$  is called the **nth Maclaurin Polynomial** for  $f$ .

$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x - c)^n = f(c) + f'(c)(x - c) + \dots + \frac{f^{(n)}(c)}{n!}(x - c)^n + \dots$  is called the **Taylor series** for  $f(x)$  at  $c$ .

Moreover, if  $c = 0$ , then the series is the **Maclaurin series** for  $f$ .

Each Term of a Taylor Series has 3 parts:

1.  $n^{\text{th}}$  Derivative of  $f(x)$  evaluated at  $c$
2.  $(x - c)^n$
3.  $n!$

$n$  goes from 0 to infinity

The  $0^{\text{th}}$  derivative is the function itself

"We build a Taylor Series Term-By-Term"

"We build each term Part-By-Part"

$$\frac{f^{(n)}(c)}{n!}(x - c)^n$$

### Alternating Polynomial Error

The Error when the Convergent Alternating Series,  $f(x) = \sum_{k=1}^{\infty} (-1)^k a_k(x)$ , is approximated by the Alternating

Finite Polynomial,  $f(x) = \sum_{k=1}^n (-1)^k a_k(x)$  for a particular value of  $x$ ,

is less than  $(a_{n+1})$ , the absolute value of the next term.

### Taylor Polynomial Error

The Error when the Taylor Series,  $\sum_{k=0}^{\infty} \frac{f^{(k)}(c)(x - c)^k}{k!}$ , is approximated by the Taylor Polynomial,

$\sum_{k=0}^n \frac{f^{(k)}(c)(x - c)^k}{k!}$  for a particular value of  $x$ ,

is less than  $\frac{f^{(k+1)}(z)(x - c)^{k+1}}{(k + 1)!}$  where  $z$  is between  $x$  and  $c$  and gives the maximum value of  $f^{(k+1)}(z)$ .

1. Construct, **from scratch**, a 2nd Degree (3 terms) Taylor Polynomial for  $f(x) = \frac{1}{x}$ , centered at  $c = 2$

2. How close is the above approximation for  $f(2.1)$ , and is the error within the Alternator Series Error Formula?

3. Write the 3rd Degree Maclaurin polynomial for  $f(x) = e^{3x}$  in SIMPLIFIED-REDUCED Form, based on what you know about the series for  $e^x$ .

4. How close is the above approximation for  $f(0.2)$ , and is the error within the Taylor Series Error Formula?

5. Find a Taylor Poly for  $f(x) = \frac{2}{x^2}$ ,  $n = 4$ ,  $c = 2$

6. Construct from scratch the 3rd Degree Taylor Polynomial centered at 1 for  $f(x) = \sqrt{x}$