Series

A series $\sum_{n=k}^{\infty} a_n$ **Converges Conditionally** if the absolute value series $\sum_{n=k}^{\infty} |a_n|$ diverges, but the original series converges.

Note that to conclude that a series converges conditionally, you need to know two things:

1. You need to know that the absolute value series diverges (so the original series doesn't converge absolutely).
2. You need to know that the original series converges.

Example. The alternating harmonic series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$
converges by the Alternating Series Test.

If we replace each term with its absolute value, we get the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

which diverges. Therefore, the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converges conditionally.

Example. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$.

If we replace each term with its absolute value, we get $\sum_{n=1}^{\infty} \frac{1}{n^3}$. This is a p-series with p = 3 > 1 so it converges.

Therefore, the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ converges absolutely.

Example. The series $\sum_{n=1}^{\infty} \frac{\sin n}{2^n} = \frac{\sin 1}{2} + \frac{\sin 2}{2^2} + \frac{\sin 3}{2^3} + \cdots$
does not alternate. In fact, $\sin 1 \approx 0.84147, \sin 2 \approx 0.90930, \sin 3 \approx 0.14112, \sin 4 \approx -0.75680, \ldots$
Therefore, we cannot use the Alternating Series Test. On the other hand, since the series has negative terms, many convergence tests, the Integral Test, the Ratio Test, the Root Test, don't apply.

We are left to consider the absolute value series $\sum_{n=1}^{\infty} \frac{|\sin n|}{2^n}$. Since $|\sin n| \le 1 \ \forall$ n, $\frac{|\sin n|}{2^n} \le \frac{1}{2^n}$ Which is a convergent geometric series. Therefore $\sum_{n=1}^{\infty} \frac{|\sin n|}{2^n}$ converges by the Direct Comparison Test, $\sum_{n=1}^{\infty} \frac{\sin n}{2^n}$ converges absolutely, and $\sum_{n=1}^{\infty} \frac{\sin n}{2^n}$ converges.

AP Calculus BC 2 Assignment 209 Wednesday, February 11, 2015 Hour   Name ____________________

Determine the convergence or divergence of the series.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{n^6}{n^7+20}$

2. $\displaystyle\sum_{n=1}^{\infty}n^7 e^{-n}$

3. $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n!}$

4. $\displaystyle\sum_{n=0}^{\infty}e^{-n}$

5. $\displaystyle\sum_{n=1}^{\infty}\frac{4^n}{3^n-1}$

6. $\displaystyle\sum_{n=1}^{\infty}\sin\frac{(2n-1)\pi}{2}$

7. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n 2^n}{n!}$

8. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{(\ln n)^n}$

9. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1} n}{2n-1}$