Limit Comparison Test:

Suppose that $a_n > 0$, $b_n > 0$, and $\lim_{n \to \infty} \frac{a_n}{b_n} = L$, where L is finite and positive.

Then the two series $\sum a_n$ and $\sum b_n$ are **both Convergent** or **both Divergent**.

Example:

Consider the Series $\sum_{n=1}^{\infty} \frac{1}{an+b}$, where a > 0 and b > 0.

Compare with the Divergent Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n \to \infty} \frac{1/(an+b)}{1/n} = \frac{1}{a}$$

Therefore $\sum_{n=1}^{\infty} \frac{1}{an+b}$ also Diverges

Absolute Convergence:

A series $\sum_{n=1}^{\infty} a_n$ **converges absolutely** if the absolute value series $\sum_{n=1}^{\infty} |a_n|$ converges. Forcing all the terms to be positive would make it more difficult for a series to converge, since there is no cancelling with the negative terms.

If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then it converges.

Ratio Test:

Let $L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$

a. If L < 1, then the series $\sum a_n$ Converges Absolutely, therefore it Converges
b. If L > 1, then the series $\sum a_n$ Diverges
c. if L = 1, then the Ratio Test Fails

Root Test:

Let $L = \lim_{n \to \infty} \sqrt[n]{|a_n|} = \lim_{n \to \infty} |a_n|^{1/n}$

a. If L < 1, then the series $\sum a_n$ Converges Absolutely, therefore it Converges
b. If L > 1, then the series $\sum a_n$ Diverges
c. if L = 1, then the Root Test Fails

Determine the convergence or divergence of: $\sum_{n=1}^{\infty} \frac{5n+3}{n^2-2n+5}$

Limit Comparison Test: Compare with the divergent harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n \to \infty} \left[\frac{5n+3}{n^2-2n+5} \div \frac{1}{n} \right] = \lim_{n \to \infty} \left[\frac{n(5n+3)}{n^2-2n+5} \right] = 5 \to \sum_{n=1}^{\infty} \frac{5n+3}{n^2-2n+5} \text{ Also } \boxed{\text{Diverges}}.$$

Determine the convergence or divergence of: $\sum_{n=0}^{\infty} \frac{n!}{3^n}$

Ratio Test: $$\lim_{n \to \infty} \left| \frac{(n+1)!}{3^{n+1}} \div \frac{n!}{3^n} \right| = \lim_{n \to \infty} \left| \frac{3^n (n+1)!}{n! \cdot 3^{n+1}} \right| = \lim_{n \to \infty} \left| \frac{(n+1)}{3} \right| \to \infty > 1 \to \sum_{n=0}^{\infty} \frac{n!}{3^n} \boxed{\text{Diverges}}$$

Determine the convergence or divergence of: $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

Root Test: $$\lim_{n \to \infty} \sqrt[n]{\left| \frac{1}{(\ln n)^n} \right|} = \lim_{n \to \infty} \frac{1}{\ln n} = 0 < 1 \to \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n} \boxed{\text{Converges}}.$$

AP Calculus BC 2 Assignment 208 Tuesday, February 10, 2015 Hour Name ______________________________

Show that each Series is Convergent or Divergent.

1. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$
2. $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$
3. $\sum_{n=1}^{\infty} \frac{2}{3^n - 5}$
4. $\sum_{n=1}^{\infty} \frac{n}{(n+1)\,2^{n-1}}$

5. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

6. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

7. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$

8. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$