The Direct Comparison Test:

1. Consider a known Convergent Series $\sum_{n=0}^{\infty} a_n$.

If Series $\sum_{n=0}^{\infty} b_n$ has each term $b_n < a_n$, then $\sum_{n=0}^{\infty} b_n$ is also Convergent.

2. Consider a known Divergent Series $\sum_{n=0}^{\infty} a_n$.

If Series $\sum_{n=0}^{\infty} b_n$ has each term $b_n > a_n$, then $\sum_{n=0}^{\infty} b_n$ is also Divergent.

The Integral Test:

If f is positive, continuous, and descending for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either **both converge** or **both diverge**.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

Let $f(x) = \frac{x}{x^2+1}$. This function is **positive**, **continuous**, and **decreasing** on $x > 1$

Use The Integral Test: $\int_1^{\infty} \frac{x}{x^2+1}dx = \lim_{a \to \infty} \int_1^{a} \frac{x}{x^2+1}dx = \frac{1}{2} \lim_{a \to \infty} \left[\ln\left(x^2+1\right)\right]_1^a$

$= \frac{1}{2} \lim_{a \to \infty} \left[\ln\left(a^2+1\right) - \ln 1\right] \to \infty \to$ The Series is $\boxed{\text{Divergent}}$.

The Harmonic Series: is $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

This qualifies for the Integral Test: $\lim_{a \to \infty} \int_1^a \frac{dx}{x} = \lim_{a \to \infty} \ln a \to \infty \to$ $\boxed{\text{The Harmonic Series Diverges}}$.

The p-Series: is: $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$

Consider situations where p < 1, where p = 0, and where p > 0.

1. Let p > 1. Let $f(x) = \frac{1}{x^p}$ This qualifies for the Integral Test. $\int_1^{\infty} \frac{1}{x^p}dx = \lim_{a \to \infty} \int_1^a \frac{1}{x^p}dx$

$= \lim_{a \to \infty} \left[\frac{1}{-p+1} * x^{-p+1}\right]_1^a = \lim_{a \to \infty} \left[\frac{1}{-p+1} * a^{-p+1} - \frac{1}{-p+1} * 1^{-p+1}\right]$

$= \lim_{a \to \infty} \left[\frac{1}{-p+1} * \frac{1}{a^{p-1}} - \frac{1}{-p+1} * 1^{-p+1}\right] = 0 + \frac{1}{p-1} \to$ $\boxed{\text{Convergent}}$

2. Let p = 1. Then we get the Harmonic Series $\to$ Divergent
3. Let p < 1. Then each term > corresponding terms of the harmonic series $\to$ $\boxed{\text{Divergent}}$

nth Term Test for Divergence:

Consider the Series $\sum_{n=0}^{\infty} a_n$. A necessary condition for convergence of a series is $\lim_{n \to \infty} a_n = 0$.

Therefore, if $\lim_{n \to \infty} a_n \neq 0$, The series Diverges.

Telescopic Series Test:

A telescopic series with the limit of the nth term approaching zero is Convergent. Moreover, the Limit can **usually** be found in the telescopic series that we are exposed to.

Alternating Series Test:

A Series with the **absolute value** of terms **approaching zero** and with **alternating signs** is Convergent.

AP Calculus BC Assignment 207 Monday, February 9, 2015 Hour Name

Show that each Series is Convergent or Divergent.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$

2. $\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n$

3. $\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26} + \frac{1}{37} + \frac{1}{50} + \cdots$

4. $\sum_{n=1}^{\infty} \frac{n+10}{10n+1}$

5. $\sum_{n=1}^{\infty} \frac{2}{3n+5}$

6. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

7. $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n-1}}$

8. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$