A **Direction Angle** of a Vector is the angle the vector makes with the positive x-axis where positive is counter-clockwise rotation.

The **Orthogonal Projection** of **u** onto **v** is: $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}.\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$. This is a Vector.

The **Absolute Value** (Magnitude) of the Orthogonal projection of u ont v is: $\|\text{proj}_{\mathbf{v}}\mathbf{u}\| = \dfrac{\mathbf{u}.\mathbf{v}}{\|\mathbf{v}\|}$. This is a Scalar.

The **Work W** done by a constant force **F** at its point of application that moves along the vector $\overrightarrow{PQ}$ can be found using either of the following:

1. $W = \|\text{proj}_{\overrightarrow{PQ}}\mathbf{F}\| \; \|\overrightarrow{PQ}\|$ (Projection Form)
2. $W = \mathbf{F} \bullet \overrightarrow{PQ}$ (Dot Product Form)

Note that Work is a Scalar, not a Vector.

Exercises:

1. **p** has magnitude 1, and a Direction Angle of 60°. Write **p** in component form.
   p = <cos 60°, sin 60°> = $\boxed{\left\langle \dfrac{1}{2}, \dfrac{\sqrt{3}}{2} \right\rangle}$
2. **u** = <6, 11>, **v** = <3, 2> . Find the projection of **u** onto **v**.
   $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\dfrac{18+22}{13}\right) < 3, 2 > = \dfrac{40}{13} <3, 2> = \boxed{\left\langle \dfrac{120}{13}, \dfrac{80}{13} \right\rangle}$
3. **u** = <0, -5000>, **v** is a Unit Vector with a Direction Angle of 30°. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.
   v = <cos 30°, sin 30°> = $\left\langle \dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right\rangle$.
   $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\dfrac{-2500}{1}\right)\left\langle \dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right\rangle = \boxed{<-1250\sqrt{3}, -1250>}$
4. **u** = <3, 5>, **v** = <-2, 7>. Find the absolute value of the projection of **v** onto **u**.
   $\|\text{proj}_{\mathbf{u}}\mathbf{v}\| = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|} = \dfrac{-6+35}{\sqrt{9+25}} = \dfrac{29}{\sqrt{34}} = \boxed{\dfrac{29\sqrt{34}}{34} \approx 4.973}$
5. A 500 pound cart is on an incline of 18°. Find the amount of force needed to prevent the cart from rolling down the incline.
   We must find the magnitude of the projection of the gravitational force onto the incline. The gravitational vector is **g** = <0, -500>. A unit vector in the direction of the incline is **p** = <cos 18°, sin 18°>. $\|\text{Proj}_{\mathbf{p}}\mathbf{g}\| =$
   $\dfrac{\mathbf{g}\cdot\mathbf{p}}{\|\mathbf{p}\|}$ = 500 sin 18° = $\boxed{\text{154.508 pounds}}$.
6. Find the <u>Work</u> performed when a force **f** of 15 pounds at an angle of 40° is used to move an object from P(2 ft, 4 ft) to Q(5 ft, 8 ft).
   f = <15 cos 40°, 15 sin 40°> & **PQ** = <3, 4>. To find the work, we compute the dot product.
   f • PQ = 45 cos 40° + 60 sin 40° = $\boxed{\text{73.039 ft-pounds}}$.
7. Find the performed when a force of 5 newtons at an angle of 22° is used to pull a wagon 28 meters along a level surface.
   F = <5 cos 22°, 5 sin 22°> & **M** = <28, 0>. **F · M** = 140 cos 22° = $\boxed{\text{129 newton meters = 129 joules}}$.

Pre-Calculus 2 Assignment 206 Friday, February 6, 2015 Hour   Name ______________________________

Find the projection of $\vec{u}$ onto $\vec{v}$. Then write u as the sum of two orthogonal vectors, one of which is $proj_v u$.

1. $\vec{u} = <3, 4>$, $\vec{v} = <8, 2>$

2. $\vec{u} = <4, 2>$, $\vec{v} = <1, -2>$

3. $\vec{u} = <0, 3>$, $\vec{v} = <2, 15>$

4. $\vec{u} = -5, -1>$, $\vec{v} = <-1, 1>$

Find the work done in moving a particle from P to Q if the magnitude and direction of the force are given by $\vec{v}$.

5. P(0, 0), Q(4, 7), $\vec{v} = <1, 4>$

6. P(1, 3), Q(-3, 5), $\vec{v} = -2\mathbf{i} + 3\mathbf{j}$

7. A truck weighing 30,000 pounds is on an incline of 5°. Find the force necessary to prevent the truck from rolling down the incline.