Infinite Series

Objectives are to explore the following:

1. Convergence vs Divergence
2. Arighmetic Series
3. n^{th} Term Test for Divergence
4. Geometric Series Test
5. Direct Comparison Test
6. Integral Test
7. p-Series Test
8. Limit Comparison Test
9. Alternating Series Test
10. Telescoping Series Test
11. Ratio Test
12. Root Test

Caution: Knowing only a few of the first terms of a sequence cannot give absolute knowledge of the rest of the terms. We can only guess as to what may be a possibility.

$$\{a_n\} : \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{2^n}, \ldots$$

$$\{b_n\} : \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{15}, \ldots, \frac{6}{(n+1)(n^2-n+6)}, \ldots$$

$$\{c_n\} : \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{7}{62}, \ldots, \frac{n^2 - 3n + 3}{9n^2 - 25n + 18}, \ldots$$

$$\{d_n\} : \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 0, \ldots, \frac{-n(n + 1)(n - 4)}{6(n^2 + 3n - 2)}, \ldots$$

Series as a sequence of partial sums.

The nth partial sum is $S_n = a_1 + a_2 + \ldots + a_n$.

If the sequence of partial sums $\{S_n\}$ converges to S, then the series $\sum a_n$ converges to S, the sum of the series.

$S = a_1 + a_2 + \ldots + a_n + \ldots$

1. If $\{S_n\}$ diverges, then the series diverges.

2. **Geometric Series Test:** An Infinite Geometric Series $a + ar + ar^2 + ar^3 + ar^4 + \ldots$ Converges iff $|r| < 1$

In this case, The Sum $= \frac{a}{1 - r}$

Find the Sum of the Infinite Series:

1. $0.1 + 0.01 + 0.001 + 0.0001 + \ldots..$ $\qquad \frac{0.1}{1 - \frac{1}{10}} = \frac{0.1}{0.9} = \boxed{\frac{1}{9}}$

2. $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \ldots.$ $\qquad \frac{0.3}{0.9} = \boxed{\frac{1}{3}}$

3. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots.$ $\qquad \frac{1}{\frac{1}{2}} = \boxed{2}$

4. $\frac{15}{100} + \frac{15}{10000} + \frac{15}{100000} + \ldots.$ $\qquad \frac{\frac{15}{100}}{\frac{99}{100}} = \boxed{\frac{5}{33}}$

Series as a Sequence of Partial Sums

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots.$$

$$S_1 = \frac{1}{2}$$

$$S_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$$

$$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = \frac{2^n - 1}{2^n}$$

$$\lim_{n \to \infty} \frac{2^n - 1}{2^n} = 1$$

$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots$ is given by $S_n = 1 - \frac{1}{n+1}$ which converges to 1

The above **Convergent Series** is an example of a **Telescopic Series**.

$\sum_{n=1}^{\infty} 1 = 1 + 1 + 1 + 1 + \cdots$ **Diverges** because $S_n = n$, the sequence of partial sums diverges.

A condition for $\sum_{n=k}^{\infty} a_n$ to be a convergent series is that $\{a_n\}$ converges to 0,
however this condition is not sufficient.

Therefore, if $\{a_n\}$ does not converge or converges to a value other than 0,
Then the series is divergent. This is the **nth Term Test for Divergence**.

Geometric Series Test: The series $\sum_{n=k}^{\infty} ar^n$ is a geometric series, and therefore **converges iff $|r| < 1$**.

The Integral Test: If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$,

then $\sum_{n=k}^{\infty} a_n$ and $\int_k^{\infty} f(x)\,dx$ either **both converge** or **both diverge**.

AP Calculus BC 2 Assignment 206 Friday, February 6, 2015 Hour Name ____________________

Determine the Convergence or Divergence of each Series. Explain your Reasoning.

1. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

2. $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

3. $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

4. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

5. $\sum_{n=1}^{\infty} 2\left(\frac{3}{4}\right)^n$

6. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$

7. $\sum_{n=1}^{\infty} \frac{1}{n^2}$