

When a vector has a magnitude of 1, it is called a **Unit Vector**.

Vector  $\hat{i}$  is **The Unit vector in the Positive x Direction**.

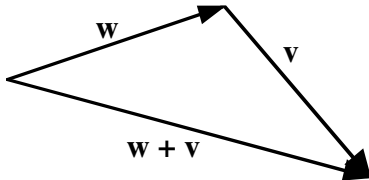
Vector  $\hat{j}$  is **The Unit vector in the Positive y Direction**.

Vector  $\hat{k}$  is **The Unit vector in the Positive z Direction**.

Basic Vector Operations:

**Vector Addition** is performed by adding the horizontal components and adding the vertical components.

$\langle 3, -4 \rangle + \langle -5, 1 \rangle = \langle -2, -3 \rangle$  Graphically, to find  $\mathbf{w} + \mathbf{v}$ , take the Tail of one  $\mathbf{v}$  and place it at the Head of  $\mathbf{w}$ , then draw a vector from the the Tail of  $\mathbf{w}$  to the head of  $\mathbf{v}$ .



The **Scalar Multiple** of the Real Number -3 times the vector:  $-3\langle 3, -5 \rangle = \langle -9, 15 \rangle$ .

Vectors  $\mathbf{u}$  and  $\mathbf{v}$  are **orthogonal** if  $\mathbf{u} \cdot \mathbf{v} = 0$

Dot Product:

$$\langle a, b \rangle \cdot \langle c, d \rangle = \langle ac, bd \rangle$$

$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$  The Dot Product is a commutative operation.

$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$  Knowing a Dot Product can help find the Angle Between 2 Vectors.

If  $\mathbf{u} \cdot \mathbf{v} = 0$ , then the vectors are orthogonal (perpendicular), and vice versa.

$$\mathbf{u} = \langle 3, -5 \rangle, \mathbf{v} = \langle 5, 7 \rangle$$

1. Find  $\mathbf{u} \cdot \mathbf{v}$

$$15 - 35 = \boxed{-20}$$

2. Find  $\|\mathbf{u}\|$

$$\sqrt{3^2 + (-5)^2}$$

$$\boxed{\sqrt{34}}$$

3. Find  $\|\mathbf{v}\|$

$$\sqrt{5^2 + 7^2}$$

$$\boxed{\sqrt{74}}$$

4. Find the angle between  $\mathbf{u}$  and  $\mathbf{v}$  in degrees.

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta,$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

$$\theta = \arccos \left( \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \right)$$

$$\theta = \arccos \left( \frac{-20}{\sqrt{(34)(74)}} \right)$$

$$\theta = \boxed{113.499^\circ}$$

Exer. 1-4: Use the vectors  $\mathbf{u} = \langle 2, 2 \rangle$ ,  $\mathbf{v} = \langle -3, 4 \rangle$ , and  $\mathbf{w} = \langle 1, -4 \rangle$  to find the indicated quantity. State whether the result is a vector or a scalar.

1.  $\mathbf{u} \cdot \mathbf{u}$
2.  $\mathbf{v} \cdot \mathbf{w}$
3.  $\mathbf{u} \cdot 2\mathbf{v}$
4.  $(\mathbf{u} \cdot 2\mathbf{v})\mathbf{w}$

Exer. 5-10: Use vectors  $\mathbf{u} = \langle 2, 2 \rangle$ ,  $\mathbf{v} = \langle -3, 4 \rangle$ , and  $\mathbf{w} = \langle 1, -4 \rangle$  to find the indicated quantity.

5.  $\mathbf{u} \cdot \mathbf{u}$
6.  $\mathbf{v} \cdot \mathbf{w}$
7.  $\mathbf{u} \cdot 2\mathbf{v}$
8.  $4\mathbf{u} \cdot \mathbf{v}$
9.  $(3\mathbf{w} \cdot \mathbf{v})\mathbf{u}$
10.  $(\mathbf{u} \cdot 2\mathbf{v})\mathbf{w}$

Exer. 11-16: Find the magnitude of  $\mathbf{u}$ .

11.  $\mathbf{u} = \langle -5, 12 \rangle$
12.  $\mathbf{u} = \langle 2, -4 \rangle$
13.  $\mathbf{u} = 20\mathbf{i} + 25\mathbf{j}$
14.  $\mathbf{u} = 6\mathbf{i} - 10\mathbf{j}$
15.  $\mathbf{u} = -4\mathbf{j}$
16.  $\mathbf{u} = 9\mathbf{i}$

Exer. 17-24: Find the angle  $\theta$  (in degrees 17-22, in radians 23-24) between the 2 vectors.

17.  $\mathbf{u} = \langle -1, 0 \rangle$ ,  $\mathbf{v} = \langle 0, 2 \rangle$
18.  $\mathbf{u} = \langle 4, 4 \rangle$ ,  $\mathbf{v} = \langle -2, 0 \rangle$
19.  $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$ ,  $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$
20.  $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$ ,  $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$
21.  $\mathbf{u} = 2\mathbf{i}$ ,  $\mathbf{v} = -3\mathbf{j}$
22.  $\mathbf{u} = 4\mathbf{j}$ ,  $\mathbf{v} = -3\mathbf{j}$
23.  $\mathbf{u} = \cos \frac{\pi}{3} \mathbf{i} + \sin \frac{\pi}{3} \mathbf{j}$ ,  $\mathbf{v} = \cos \frac{3\pi}{4} \mathbf{i} + \sin \frac{3\pi}{4} \mathbf{j}$
24.  $\mathbf{u} = \cos \frac{\pi}{4} \mathbf{i} + \sin \frac{\pi}{4} \mathbf{j}$ ,  $\mathbf{v} = \cos \frac{2\pi}{3} \mathbf{i} + \sin \frac{2\pi}{3} \mathbf{j}$

Exer. 25-26: Find  $\mathbf{u} \cdot \mathbf{v}$ , where  $\theta$  is the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

25.  $\|\mathbf{u}\| = 9$   $\|\mathbf{v}\| = 16$   $\theta = \frac{3\pi}{4}$
26.  $\|\mathbf{u}\| = 4$   $\|\mathbf{v}\| = 12$   $\theta = \frac{\pi}{3}$