A205
Thursday, February 5, 2014
A Sequence is a function where the domain is the set of positive integers.

We represent the n^{th} term as a_n.
Let $a_n = \left\{\frac{n}{1-2n}\right\}$
The 1^{st} few terms are: -1, -2/3, -3/5, -4/7, ….

The terms of a sequence can be recursively defined.
$d_1 = 25 \quad d_{n+1} = d_n - 5$
The 1^{st} few terms are: 25, 20, 15, 10, ….

$\lim_{n \to \infty} a_n = L$ if for each $\varepsilon > 0, \exists M > 0 \ni |a_n - L| < \varepsilon$ whenever n > M. If the limit L exists, then the sequence converges to L. If the limit of a sequence does not exist, then the sequence diverges.

If a real function has a limit as n approaches infinity, then the corresponding sequence defined at the integers has the same limit.

Do the following sequences converge?
$\{a_n\} = \{3 + (-1)^n\}$
$\{b_n\} = \left\{\frac{n}{1-2n}\right\}$

Write a sequence $\{a_n\}$ with the following 1^{st} 5 terms: 2/1, 4/3, 8/5, 16/7, 32/9, …
$\{a_n\} = \left\{\frac{2^n}{2n-1}\right\}$

Does this sequence Converge or Diverge?
It can be shown to be Divergent, using L'Hôpital's Rule.

A sequence is Monotonic if its terms are non decreasing or non increasing.

A sequence is **Bounded Above** if all $a_n \le M$ for some real number M.
A sequence is **Bounded Below** if all $a_n \ge M$ for some real number M.
A sequence is **Bounded** if it is Bounded Below and Bounded Above

If a sequence is Bounded and Monotonic, then it converges.

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Exer. 1-3: Write the first 5 terms of the sequence.

1. $a_n = \frac{2n}{n + 3}$

2. $a_n = \frac{2n}{n + 3}$

3. $a_n = 10 + \frac{2}{n} + \frac{6}{n^2}$

Exer. 4: Write the first five terms of the recursively defined sequence.

4. $a_1 = 4,\ a_{k+1} = \left(\frac{k + 1}{2}\right) a_k$

Exer. 5: Simplify the ratio of factorials.

5. $\frac{(n + 2)!}{n!}$

Exer. 6-9: Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

6. $a_n = \frac{3n^2 - n + 4}{2n^2 + 1}$

7. $a_n = \frac{\ln n^3}{2n}$

8. $a_n = \frac{n^2}{2n + 1} - \frac{n^2}{2n - 1}$

9. $a_n = \left(1 + \frac{k}{n}\right)^n$