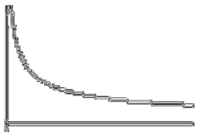
A204

Improper Integrals

An integral if the interval is infinite or if the integrand has an infinite discontinuity on the interval.



f(x) = $\frac{1}{\sqrt{x}}$

Find the area under f(x) above the x-axis from x = 0 to x = 1.

f(x) is not defined at x = 0, and therefore we call it an **improper integral**.
At first glance, it may seem that the area is infinite, but you would be wrong.

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \int_0^1 x^{-1/2}\,dx = \lim_{a\to 0}\int_a^1 x^{-1/2}\,dx = \lim_{a\to 0}\left[2\sqrt{x}\right]_a^1 = \lim_{a\to 0}\left[2\sqrt{1}-\sqrt{a}\right] = \boxed{2}$$

Because we have a finite result, this integral is convergent.

a. If f is continuous on [a, ∞), then $\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$

b. If f is continuous on [a, b) and has an infinite discontinuity at b, then $\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx$

$\int_0^1 \frac{1}{x}\,dx$ Diverges

$\int_0^1 \frac{1}{\sqrt{x}}\,dx$ Converges to 2

$\int_1^\infty \frac{1}{x^2}\,dx$ Converges to 1

$$\int_1^\infty \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1} & if\ \ p > 1 \\ Diverges\ if & p \le 1 \end{cases}$$

f(x) = $\frac{1}{x^2}$ Find the area under the curve from x = -1 to x = 1.

$$\lim_{a\to 0^-}\int_{-1}^a \frac{1}{x^2}\,dx + \lim_{b\to 0^+}\int_b^1 \frac{1}{x^2}\,dx = \lim_{a\to 0^-}\left[-\frac{1}{x}\right]_{-1}^a + \lim_{b\to 0^+}\left[-\frac{1}{x}\right]_b^1 = \text{DNE}$$

AP Calculus BC 2 Assignment 204 Wednesday, February 4, 2015 Hour Name ______________________________

1. $\int_0^{\infty} e^{-x}\,dx =$

2. $\int_1^{\infty} \frac{5}{x^3}\,dx =$

3. $\int_0^{\infty} xe^{-x/2}\,dx =$

4. $\int_1^{\infty} \frac{\ln x}{x}\,dx =$

5. $\int_0^{\infty} \frac{1}{e^x + e^{-x}}\,dx =$

6. $\int_0^{8} \frac{8}{x}\,dx =$

7. $\int_0^{6} \frac{4}{\sqrt{6-x}}\,dx =$