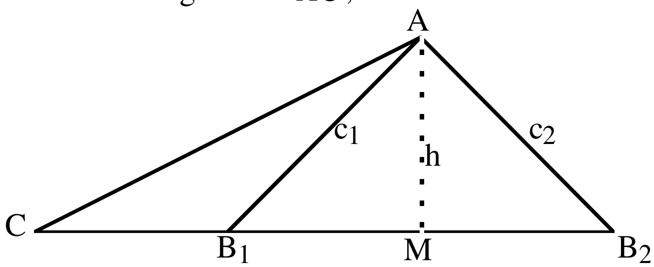
001
Ambiguous Case - SSA

When given two sides and a non-included angle of a triangle, we refer to this as SSA

In the following triangle, $\overline{AC} = 80$ and $\angle C = 48°$. The length of the side opposite $\angle C$ will determine whether we have 0 triangles, 1 triangle, or 2 triangles.
The **minimum** value for the side opposite $\angle C$ would be 80 sin C. We will call this value h, our **test value**.
In this case h = 80 sin 48° = 59.45158604.
If side c is less than h,
1. If c is less than h, then the triangle does not exist.
2. If c is equal to h, then there will be exactly one right triangle.
3. If c is larger than h, but smaller than $\overline{AC}$, then there will be 2 triangles as shown below.
4. If c is larger than $\overline{AC}$, then there will be exactly one triangle.



Example 1
Given the above information for ΔABC with $\overline{AC} = 80$ and $\angle C = 48°$. Also let c = 40.
This is the Ambiguous situation because the information is SSA.
Since c is too short, the triangle does not exist.

Example 2
Given the above information for ΔABC with $\overline{AC} = 80$ and $\angle C = 48°$. Also let c = 70.
This is the Ambiguous situation because the information is SSA.
Since c is between $\overline{AC}$ and h, then we will 2 triangles as shown above. There will be 2 solutions for $\angle B$, 2 solutions for $\angle A$, and 2 solutions for side b.
It is important to note that the LAW of SINES cannot give an answer for an angle greater than 90°. Therefore when you use the law of sines $\frac{\sin B_2}{80} = \frac{\sin 48°}{70} \rightarrow B_2 = \arcsin\left(\frac{80\sin 48°}{70}\right)$ = 58.13652361°.
$\angle B_1$ is supplementary to $\angle B_2$, so $\angle B_1$ =180° - 58.13652361°= 121.8634764°.
$\angle A_2$= 180 - $\angle B_2$- 48° = 73.86347639°.
$\angle A_1$= 180 - $\angle B_1$- 48° = 10.13652361°.
$\frac{a_2}{\sin A_2} = \frac{70}{\sin 48°} \rightarrow a_2 = \frac{70\sin A_2}{\sin 48°}$ = 90.4832416
$\frac{a_1}{\sin A_1} = \frac{70}{\sin 48°} \rightarrow a_1 = \frac{70\sin A_1}{\sin 48°}$ = 16.57765541
We will organize our results in a table like the following where Δ_2 is the larger one, and Δ_1 is the smaller one.

Δ_2	B = 58.137	A = 73.863	a = 90.483
Δ_1	B = 121.863	A = 10.137	a = 16.578

Pre-Calculus 2 Assignment 01 Monday, January 26, 2015 Hour Name ______

Solve ΔABC

1. A = 25°, B = 60°, a = 12

2. B = 15°, C = 125°, c = 20

3. A = 60°, a = 9, c = 10

4. A = 58°, a = 4.5, b = 12.8

5. A = 50°, B = 110°, b = 42

6. a = 12, b = 16, c = 18

7. a = 16, B = 40°, c = 10

8. A = 80°15', B = 25°30', b = 2.8

9. a = 12, B = 35°47', c = 16