L'Hopital's Rule:
Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly c itself. Assume that g'(x) $\neq 0\ \forall x \in (a,b)$, except possibly at c itself. If $\lim_{x \to c} \frac{f(x)}{g(x)}$ produces the indeterminate form $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, then $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$ provided the limit on the right exists (or is infinite)

1. $\lim_{x \to 2} \frac{x^2 - 6x + 8}{x - 2}$

Direct Substitution leads to the indeterminate form $\frac{0}{0}$

By L'Hopital's Rule (L'H may be used as an abbreviation):

$$\lim_{x \to 2} \frac{x^2 - 6x + 8}{x - 2} = \lim_{x \to 2} \frac{2x - 6}{1} = \boxed{-2}$$

2. $\lim_{x \to 0^+} (\tan x)^x \to 0^0$

let y = $(\tan x)^x$ Now take the limit of the natural log of both sides of the equation.

$$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \ln(\tan x)^x = \lim_{x \to 0^+} x \ln(\tan x) = \lim_{x \to 0^+} \frac{\ln(\tan x)}{\frac{1}{x}} \to \frac{-\infty}{\infty} \text{ L'H}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{\tan x}\sec^2 x}{-\frac{1}{x^2}} = \lim_{x \to 0^+} \frac{-x^2}{\sin x \cos x} \to \frac{0}{0} \text{ L'H } \lim_{x \to 0^+} \frac{-2x}{\cos^2 x - \sin^2 x} = \frac{0}{1} = 0$$

The only way for ln y = 0 is when y = 1 which means $\lim_{x \to 0^+} (\tan x)^x = \boxed{1}$.

3. $\lim_{x \to -\infty} xe^x$ L'H does not apply, so we will rewrite the expression.

$\lim_{x \to -\infty} \frac{e^x}{\frac{1}{x}} \to \frac{0}{0}$ L'H $\lim_{x \to -\infty} \frac{e^x}{-\frac{1}{x^2}} \to \frac{0}{0}$ L'H $\lim_{x \to -\infty} \frac{e^x}{\frac{2}{x^3}} \to \frac{0}{0}$ Going nowhere, so we try a different approach. A different re-write gives the following $\lim_{x \to -\infty} \frac{x}{e^{-x}} \to \frac{-\infty}{\infty}$ L'H

$$\lim_{x \to -\infty} \frac{1}{-e^{-x}} = 0$$

AP Calculus BC 1 Assignment 152 Thursday, January 7, 2016 Hour Name

#'s 1-5: Non-Calculator

1. $\int_0^3 \frac{2x^2 + x - 3}{x} \, dx =$

2. $\lim_{x \to 4} \begin{cases} e^x & 0 < x < 4 \\ \ln(2x) & 4 \le x < 12 \end{cases}$

3. $f(x) = e^{2 \sin 2x}$ $\quad f'(x) =$

4. $f(x) = x^3 \sqrt{3x^4 + 1}$ Find the average value of f(x) on [1, 3].

5. A conical pile has diameter twice that of the height. The height changes at a rate of 5/8 meters per minute. Find the rate at which the volume is changing when the height is 2 meters

#'s 6-12: Calculator Active

6. $f(x) = 2e^x$ $\quad g(x) = -x^2 + 1$ Find the value(s) of x where the tangent lines for the functions are parallel.

7. $\lim_{x \to w} \frac{x^3 - w^3}{x^4 - w^4} =$ Note: w is a constant.

8. $f'(x) = \frac{(x-1)^3}{\sin(2x)}$. Find the number of critical values of f(x) on (-3, 3)?

9. Find the trapezoidal approximation for $\int_3^9 f(x) \, dx$, using the tabular data.

x	3	5	6	9
f(x)	6	4	2	1

10. y = f(x). f(3) = 6. The slope of f is $\frac{3x^2+1}{y}$. Find the equation of the tangent line to f(x) at x = 3.

11. Use the information in #10 find the linear approximation for f(4).

12. Write the particular solution for f(x).

13. Find the actual value of f(4).

#'s 14-16: Non-Calculator

14. $\lim_{x \to 1} \frac{1 - x + \ln x}{1 + \cos \pi x} =$

15. $\lim_{x \to 0} x \ln x =$

16. $\lim_{x \to 1} \frac{2 \ln x}{x - 1} =$