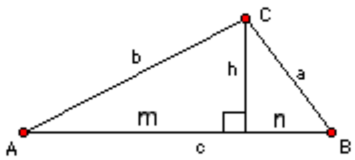
Pre-Calculus
150

1. Proof of the Law of Sines:



$\sin A = \frac{h}{b}$ $\sin B = \frac{h}{a}$

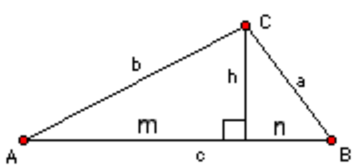
$h = b\sin A$ $h = a\sin B$

$$b\sin A = a\sin B$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Law of Sines: for ΔABC, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

2. Proof of the Law of Cosines:



$\cos A = \frac{m}{b} \rightarrow m = b\cos A$

$n = c - m = c - b\cos A$

$\sin A = \frac{h}{b} \rightarrow h = b\sin A$

$a^2 = h^2 + n^2$

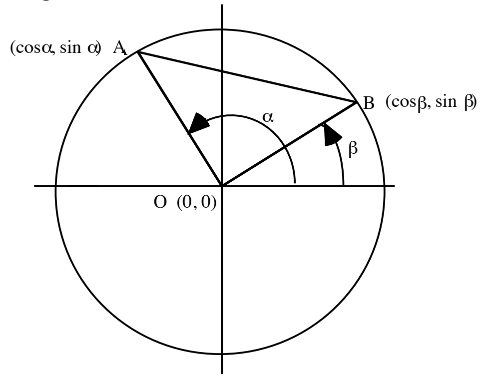
$a^2 = b^2 \sin^2 A + c^2 - 2bc\cos A + b^2\cos^2 A$

$a^2 = b^2\left(\sin^2 A + \cos^2 A\right) + c^2 - 2bc\cos A$

$a^2 = b^2 + c^2 - 2bc\cos A$

Law of Cosines: for ΔABC, $a^2 = b^2 + c^2 - 2bc\cos A$

Trigonometric Sum and Difference Theorems



Consider any 2 points, A and B, on the unit circle. The radius segments to these points form angles of measure α and measure β as shown in the figure. Point O(0, 0) is at the origin and the center of the unit circle.
Since points A and B are on the unit circle, we know their coordinates to be:
$A(\cos\alpha, \sin\alpha)$ and $B(\cos\beta, \sin\beta)$
In the triangle OB = OA = 1, since the radius of the unit circle is 1.

By the **Law of Cosines**:

$(AB)^2 = (OA)^2 + (OB)^2 - 2(OA)(OB)\cos(\alpha - \beta)$

$(AB)^2 = 1 + 1 - 2 \bullet 1 \bullet 1 \bullet \cos(\alpha - \beta)$

$(AB)^2 = 2 - 2\cos(\alpha - \beta)$

By the **Distance Formula**:

$AB = \sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2}$

$(AB)^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

$(AB)^2 = \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta$
$\quad + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta$

$(AB)^2 = 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

Since both of the above equations have $(AB)^2$ on the left, then we can say:
$2 - 2\cos(\alpha - \beta) = 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$
Subtracting 2 from both sides, then dividing both sides by -2, we get:
$\boxed{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$ Q.E.D. This is our **Basic Formula**

1. $\cos(x - y) = \cos x\cos y + \sin x\sin y$
2. $\cos(x + y) = \cos x\cos y - \sin x\sin y$
3. $\sin(x + y) = \sin x\cos y + \cos x\sin y$
4. $\sin(x - y) = \sin x\cos y - \cos x\sin y$
5. $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x\tan y}$
6. $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x\tan y}$

Double Angle Formulas:

7. $\sin 2x = 2\sin x\cos x$
8. $\cos 2x = \cos^2 x - \sin^2 x$
9. $\tan 2x = \dfrac{2\tan x}{1 + tan^2 x}$

<u>Memorize these 9 Formulas</u>

10. $\sin x = \frac{3}{5}$ $\cos y = -\frac{5}{13}$ $x \in I$ $y \in II$ Find $\sin(x + y)$

$x + y \in II$ $\cos x = \frac{4}{5}$ $\sin y = \frac{12}{13}$ $\sin(x + y) = \sin x\cos y + \cos x\sin y = \sin x\cos y + \cos x\sin y$

$= \frac{3}{5} \bullet \frac{-5}{13} + \frac{4}{5} \bullet \frac{12}{13} = \frac{-15 + 48}{65} = \frac{33}{65}$

11. Simplify: $\cos 55° \cos 25° + \sin 55° \sin 25° = (\cos 55° - 25°) = \cos 30° = \frac{\sqrt{3}}{2}$

12. Find the exact value of $\sin 75°$.

$\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30° = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$

Pre-Calculus 1 Assignment 150 Tuesday, January 5, 2016 Hour Name

05.04 Sum and Difference Formulas

Find the exact value of the sine, cosine, and tangent of the angle.

1.	$105^\circ = 60^\circ + 45^\circ$
2.	$165^\circ = 135^\circ + 30$
3.	$195^\circ = 225^\circ - 30^\circ$
4.	$255^\circ = 300^\circ - 45^\circ$
5.	$\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$
6.	$\frac{17\pi}{12} = \frac{7\pi}{6} + \frac{\pi}{4}$
7.	$-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$
8.	$-\frac{19\pi}{12} = \frac{2\pi}{3} - \frac{9\pi}{4}$
9.	75°
10.	15°
11.	-225°
12.	-165°
13.	$\frac{13\pi}{12}$
14.	$\frac{5\pi}{12}$
15.	$-\frac{7\pi}{12}$
16.	$-\frac{13\pi}{12}$