

Properties of Powers

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|------------------------|--|
| 1. Product of Powers | $(a^m)(a^n) = a^{m+n}$ |
| 2. Power of a Product | $(ab)^p = a^p b^p$ |
| 3. Power of a Power | $(a^m)^n = a^{mn}$ |
| 4. Negative Exponent | $a^{-n} = \frac{1}{a^n}$ |
| 5. Quotient of Powers | $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ |
| 6. Power of a Quotient | $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ |
| 7. Zero Exponent | $a^0 = 1, a \neq 0$ |

Simplifying Products and Quotients

- $\sqrt{3} \cdot \sqrt{12} = \sqrt{3 \cdot 12} = \sqrt{36} = 6$
- $\sqrt[3]{5} \cdot \sqrt[3]{25} = \sqrt[3]{5 \cdot 25} = \sqrt[3]{125} = 5$
- $\sqrt[4]{27} \cdot \sqrt[4]{3} = \sqrt[4]{27 \cdot 3} = \sqrt[4]{81} = 3$
- $\frac{\sqrt{98}}{\sqrt{2}} = \sqrt{\frac{98}{2}} = \sqrt{49} = 7$
- $\frac{\sqrt[4]{4}}{\sqrt[4]{1024}} = \sqrt[4]{\frac{4}{1024}} = \sqrt[4]{\frac{1}{256}} = \frac{1}{4}$
- $\frac{\sqrt[3]{625}}{\sqrt[3]{5}} = \sqrt[3]{\frac{625}{5}} = \sqrt[3]{125} = 5$
- $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5} = 3\sqrt[3]{5}$
- $\frac{\sqrt[5]{7}}{\sqrt[5]{8}} = \frac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \frac{\sqrt[5]{4}}{\sqrt[5]{4}} = \frac{\sqrt[5]{28}}{\sqrt[5]{32}} = \frac{\sqrt[5]{28}}{2}$

$a\sqrt{b} + c\sqrt{d}$ & $a\sqrt{b} - c\sqrt{d}$ are **conjugates** of each other.

When a denominator is a sum or a difference involving square roots, we multiply both the numerator and the denominator by the **conjugate** of the denominator.

- $\frac{1}{5 + \sqrt{3}} = \frac{1}{5 + \sqrt{3}} \cdot \frac{5 - \sqrt{3}}{5 - \sqrt{3}} = \frac{5 - \sqrt{3}}{5^2 - (\sqrt{3})^2} = \frac{5 - \sqrt{3}}{22}$
- $\frac{1}{\sqrt{7} - 2} = \frac{1}{\sqrt{7} - 2} \cdot \frac{\sqrt{7} + 2}{\sqrt{7} + 2} = \frac{\sqrt{7} + 2}{(\sqrt{7})^2 - 2^2} = \frac{\sqrt{7} + 2}{3}$

Radical expressions with the same index and radicand are **like radicals**. To add or subtract like radicals, we use the **distributive property** as we normally did with like terms.

1. $\sqrt[4]{10} + 7\sqrt[4]{10} = (1 + 7)\sqrt[4]{10} = 8\sqrt[4]{10}$
2. $5(7^{1/3}) + 6(7^{1/3}) = (5 + 6)(7^{1/3}) = 11(7^{1/3})$
3. $2\sqrt[6]{17} + 9\sqrt[6]{17} = (2 + 9)\sqrt[6]{17} = 11\sqrt[6]{17}$
4. $\sqrt[4]{48} - \sqrt[4]{3} = \sqrt[4]{16 \cdot 3} - \sqrt[4]{3} = 2\sqrt[4]{3} - \sqrt[4]{3} = (2 - 1)\sqrt[4]{3} = \sqrt[4]{3}$
5. $\sqrt[3]{54} - \sqrt[3]{2} = \sqrt[3]{27 \cdot 2} - \sqrt[3]{2} = 3\sqrt[3]{2} - \sqrt[3]{2} = (3 - 1)\sqrt[3]{2} = 2\sqrt[3]{2}$

Simplify the Following

1. $\sqrt[4]{27} \cdot \sqrt[4]{3} = \sqrt[4]{81} = 3$
2. $2(8^{1/5}) + 10(8^{1/5}) = 12(8^{1/5})$
3. $\frac{\sqrt[3]{250}}{\sqrt[3]{2}} = \sqrt[3]{125} = 5$
4. $\sqrt[3]{104} = \sqrt[3]{8 \cdot 13} = 2\sqrt[3]{13}$
5. $\sqrt[5]{\frac{3}{4}} = \sqrt[5]{\frac{3}{4}} \cdot \sqrt[5]{\frac{8}{8}} = \sqrt[5]{\frac{24}{32}} = \frac{\sqrt[5]{24}}{2}$
6. $4(9^{2/3}) + 8(9^{2/3}) = 12(9^{2/3}) = 12\sqrt[3]{81} = 12\sqrt[3]{27 \cdot 3} = 12 \cdot 3\sqrt[3]{3} = 36\sqrt[3]{3}$

Simplify the following.

1. $(9^2)^{1/3} =$

2. $(12^2)^{1/4} =$

3. $\frac{6}{6^{1/4}} =$

4. $\frac{7}{7^{1/3}} =$

5. $\left(\frac{9^3}{6^3}\right)^{-1/3} =$

6. $\frac{49^{3/8} \cdot 49^{7/8}}{7^{5/4}} =$

7. $\frac{11}{9 - \sqrt{6}} =$

8. $\frac{2}{\sqrt{8} + \sqrt{7}} =$

9. $\sqrt[3]{16} \cdot \sqrt[3]{32} =$

10. $\frac{\sqrt[3]{6} \cdot \sqrt[3]{72}}{\sqrt[3]{2}} =$

11. $\frac{\sqrt[3]{3} \cdot \sqrt[3]{18}}{\sqrt[6]{2} \cdot \sqrt[6]{2}} =$

12. $5\sqrt{12} - 19\sqrt{3} =$

13. $\sqrt[5]{224} + 3\sqrt[5]{7} =$

14. $5(24^{1/3}) - 4(3^{1/3}) =$