Pre-Calculus
149

Trigonometric Equations

Solve the following:

$2\sin x - 1 = 0$ Solve on $\Re$
$2\sin x = 1$
$\sin x = ½$

$$\boxed{x = \frac{\pi}{6} + 2n\pi \text{ and } x = \frac{5\pi}{6} + 2n\pi}$$

$\sin(x) + \sqrt{2} = -\sin(x)$ Solve on $\Re$
$2\sin(x) = -\sqrt{2}$
$\sin(x) = -\frac{\sqrt{2}}{2}$

$$\boxed{x = \frac{5\pi}{4} + 2n\pi \text{ and } x = \frac{7\pi}{4} + 2n\pi}$$

$3\tan^2 x - 1 = 0$ Solve on $\Re$
$3\tan^2 x = 1$
$\tan^2 x = \frac{1}{3}$
$\tan x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$$\boxed{x = \frac{\pi}{6} + n\pi \text{ and } x = \frac{5\pi}{6} + n\pi}$$

$\cot x \cos^2 x = 2\cot x$ Solve on $\Re$
$\cot x \cos^2 x - 2\cot x = 0$
$\cot x(\cos^2 x - 2) = 0$
$\cot x = 0$ and $\cos^2 x = 2$
$\cot x = 0$ and $\cos x = \pm\sqrt{2}$ Not possible
$x = \frac{\pi}{2} + n\pi$

$2\sin^2 x - \sin x - 1 = 0$ Solve on the interval $[0, 2\pi)$
$(2\sin x + 1)(\sin x - 1) = 0$
$\sin x = -1/2$ and $\sin x = 1$

$$\boxed{x = \frac{7\pi}{6} \; and \; \frac{11\pi}{6} \quad x = \frac{\pi}{2}}$$

$2\sin^2 x + 3\cos x - 3 = 0$ Solve on $\Re$

$2(1 - \cos^2 x) + 3\cos x - 3 = 0$

$-2\cos^2 x + 3\cos x - 1 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = ½$ and $\cos x = 1$

$$\boxed{x = \frac{\pi}{3} + 2n\pi \text{ and } \frac{5\pi}{3} + 2n\pi \text{ and } 2n\pi}$$

$\cos x + 1 = \sin x$ Solve on the interval $[0, 2\pi)$

$\cos^2 x + 2\cos x + 1 = \sin^2 x$ Square Both Sides

$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$\cos x = 0$ and $\cos x = -1$

$x = \frac{\pi}{2}\ and\ \frac{3\pi}{2}\ and\ \pi$

CAUTION: Whenever we square both sides, we introduce the possibility of extraneous answers. Therefore we must check the solutions in the original equation.

$\cos\left(\frac{\pi}{2}\right) + 1 = 1, \sin\left(\frac{\pi}{2}\right) = 1$ Check

$\cos\left(\frac{3\pi}{2}\right) + 1 = 1, \sin\left(\frac{3\pi}{2}\right) = -1$ Does Not Check

$\cos(\pi) + 1 = 0, \sin(\pi) = 0$ Check

$$\boxed{x = \frac{\pi}{2}\ and\ \pi}$$

$2\cos 3t - 1 = 0$ Solve on $\Re$

$\cos(3t) = ½$

$3t = \frac{\pi}{3} + 2n\pi\ and\ \frac{5\pi}{3} + 2n\pi$

$$\boxed{t = \frac{\pi}{9} + \frac{2n\pi}{3}\ and\ \frac{5\pi}{9} + \frac{2n\pi}{3}}$$

$3\tan^2 x = 5\sec x - 1$ Solve on $\Re$

$3(\sec^2 x - 1) = 5\sec x - 1$

$3\sec^2 x - 3 = 5\sec x - 1$

$3\sec^2 x - 5\sec x - 2 = 0$

$(3\sec x + 1)(\sec x - 2) = 0$

$\sec x = -\frac{1}{3}\quad and\quad \sec x = 2$

The only valid statement is sec x = 2

$$\boxed{x = \frac{\pi}{3} + 2n\pi\quad and\quad x = \frac{5\pi}{3} + 2n\pi}$$

$\sec^2 x - 2\tan x = 4$ Solve on $\Re$
$\tan^2 x + 1 - 2\tan x - 4 = 0$
$\tan^2 x - 2\tan x - 3 = 0$
$(\tan x - 3)(\tan x + 1) = 0$
$\tan x = 3$ and $\tan x = -1$

$$\boxed{x = \arctan 3 + n\pi \quad \text{and} \quad x = \frac{3\pi}{4} + n\pi}$$

$\sin x = 6\cos^2 x$ Solve on $\Re$
$\sin x = 6(1 - \sin^2 x)$
$\sin x = 6 - 6\sin^2 x$
$6\sin^2 x + \sin x - 6 = 0$
In this case factoring is not possible, so we must use the Quadratic Formula
$a = 6, b = 1, c = -6$

$$\sin x = \frac{-1 \pm \sqrt{1 - 4(6)(-6)}}{12} = \frac{-1 \pm \sqrt{145}}{12}$$

$\sin x = \dfrac{-1 - \sqrt{145}}{12}$ is **<u>not valid</u>** because it is less than -1

Therefore $\sin x = \dfrac{-1 + \sqrt{145}}{12}$

$$\boxed{x = \arcsin\left(\frac{-1 + \sqrt{145}}{12}\right) + 2n\pi \quad \text{and} \quad x = \frac{\pi}{2} - \arcsin\left(\frac{-1 + \sqrt{145}}{12}\right) + 2n\pi}$$

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Find all solutions of the equation in the interval $[0, 2\pi)$ algebraically.

1. $2 \sin x + 1 = 0$

2. $\tan^2 x - 1 = 0$

3. $2 \sin^2 x = 2 + \cos x$

4. $\sec x \csc x = 2 \csc x$

5. $\sec x + \tan x = 1$

6. $\sin^2 x + \cos x + 1 = 0$

7. $2 \cos^2 x + \cos x - 1 = 0$

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Find all solutions of the equation in the interval $[0, 2\pi)$ algebraically.

1.	$2 \sin x + 1 = 0$	$\sin x = -\frac{1}{2}$ $\boxed{x = \frac{7\pi}{6} \quad \frac{11\pi}{6}}$
2.	$\tan^2 x - 1 = 0$	$\tan x = \pm 1$ $x \boxed{= \frac{\pi}{4} \; \frac{3\pi}{4} \; \frac{5\pi}{4} \; \frac{7\pi}{4}}$
3.	$2 \sin^2 x = 2 + \cos x$	$2 - 2 \cos^2 x = 2 + \cos x$ $\quad 2 \cos^2 x + \cos x = 0$ $\cos x(2 \cos x + 1) = 0 \rightarrow \cos x = 0 \;\; \cos x = -\frac{1}{2}$ $\boxed{x = \frac{\pi}{2} \; \frac{3\pi}{2} \; \frac{2\pi}{3} \; \frac{4\pi}{3}}$
4.	$\sec x \csc x = 2 \csc x$	$\sec x \csc x - 2 \csc x = 0$ $\quad \csc x(\sec x - 2) = 0 \rightarrow \csc x = 0 \;\; \sec x = 2$ $\boxed{x = \frac{\pi}{3} \; \frac{5\pi}{3}}$
5.	$\sec x + \tan x = 1$	$\frac{1}{\cos x} + \frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} = 0$ $\quad 1 + \sin x - \cos x = 0$ $\quad 1 + \sin x = \cos x$ $1 + 2 \sin x + \sin^2 x = \cos^2 x$ $\quad 1 + 2 \sin x + \sin^2 x = 1 - \sin^2 x$ $2 \sin^2 x + 2 \sin x = 0$ $\quad \sin x(\sin x + 1) = 0$ $\quad x = 0 \;\; \pi \;\; \frac{3\pi}{2}$ $\boxed{x = 0}$.
6.	$\sin^2 x + \cos x + 1 = 0$	$1 - \cos^2 x + \cos x + 1 = 0$ $\quad \cos^2 x - \cos x - 2 = 0$ $(\cos x - 2)(\cos x + 1) = 0$ $\boxed{x = \pi}$
7.	$2 \cos^2 x + \cos x - 1 = 0$	$(2 \cos x - 1)(\cos x + 1) = 0$ $\boxed{x = \frac{\pi}{3} \; \frac{5\pi}{3} \; \pi}$