Calculus BC
149

Partial Fraction Decomposition

Decomposition of $\frac{N(x)}{D(x)}$ into Partial Fractions

1. If the degree of N(x) > degree of D(x), perform Long Division. This gives an expression of several terms some of which may decompose into partial fractions.
2. Completely Factor the Denominator into factors of the form $(px + q)^m$ and $(ax^2 + bx + c)^n$ where $ax^2 + bx + c$ is irreducible.
3. For each Linear Factor of the form $(px + q)^m$, the partial fraction decomposition must include the following sum of m fractions:
$$\frac{A_1}{(px+q)}+\frac{A_2}{(px+q)^2}+\ldots+\frac{A_m}{(px+q)^m}$$
4. For each Quadratic Factor of the form $(ax^2 + bx + c)^n$, the partial fraction decomposition must include the following sum of n fractions:
$$\frac{B_1x+C_1}{ax^2+bx+c}+\frac{B_2x+C_2}{(ax^2+bx+c)^2}+\ldots+\frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

$$\int\frac{3x-1}{x^2-1}dx=$$
$$\frac{3x-1}{x^2-1}=\frac{3x-1}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$$
$$3x-1=A(x-1)+B(x+1)$$
Let x = -1 $\rightarrow$ -4 = -2A $\rightarrow$ A = 2
Let x = 1 $\rightarrow$ 2 = 2B $\rightarrow$ B = 1
$$\frac{3x-1}{x^2-1}=\frac{2}{x+1}+\frac{1}{x-1}$$
$$\int\frac{3x-1}{x^2-1}dx=\int\left(\frac{2}{x+1}+\frac{1}{x-1}\right)dx$$
$$\boxed{2\ln|x+1|+\ln|x-1|+C}$$

$$\int\frac{1}{x^2+5x+6}dx=$$
$$\frac{1}{x^2+5x+6}=\frac{A}{x+2}+\frac{B}{x+3}$$
1 = A(x + 3) + B(x + 2)
Let x = -2 $\rightarrow$ 1 = A $\rightarrow$ A = 1
Let x = -3 $\rightarrow$ 1 = -B $\rightarrow$ B = -1
$$\int\frac{1}{x^2+5x+6}dx=\int\left(\frac{1}{x+2}-\frac{1}{x+3}\right)dx=\boxed{\ln|x+2|-\ln|x+3|+C}$$

$$\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\,dx$$

$x^3 + 2x^2 + x = x(x^2 + 2x + 1) = x(x + 1)^2$

$$\frac{5x^2 + 20x + 6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Multiply Both Sides by the Common Denominator:

$5x^2 + 20x + 6 = A(x + 1)^2 + Bx(x + 1) + Cx$

Let $x = 0 \rightarrow 6 = A \rightarrow A = 6$

Let $x = -1 \rightarrow -9 = -C \rightarrow C = 9$

To find B, choose any other value for x:

Let $x = 1 \rightarrow 31 = 6(4) + 2B + 9 \rightarrow 31 = 2B + 33 \rightarrow B = -1$

$$\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\,dx = \int \left( \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2} \right) dx$$

$$= \boxed{6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C}$$

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Non- Calculator

1. $\int \frac{1}{x^2 - 1}\,dx =$

2. $\int \frac{5 - x}{2x^2 + x - 1}\,dx =$

3. $\int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8}\,dx =$

4. $\displaystyle\int \frac{x^2+3x-4}{x^3-4x^2+4x}\,dx=$

5. $\displaystyle\int \frac{x^2}{x^4-2x^2-8}\,dx=$