AP Calculus BC
148

Euler's Method: For Approximating the value of a function when an initial condition is given along with a differential equation.

If we are given a point (x_0, y_0) on a straight line with slope m, we can find the y coordinate y_1, of a point whose x-coordinate x_1 is Δx units to the right of x_0.

Since $m = \frac{y_1 - y_0}{\Delta x}$ We can get $y_1 - y_0 = m\ \Delta x$. This leads to $\boxed{y_1 = y_0 + \Delta x(m)}$.

Euler's Method is a calculus interpretation of this notion.

Given a Differential Equation where we can isolate $\frac{dy}{dx}$.
Given a particular solution (x_0, y_0).
Given a Step Value Δx

Translation to Calculus: $\boxed{y_1 = y_0 + \Delta x \left.\frac{dy}{dx}\right|_{x=x_0}}$. This is the 1st Iteration.

To go through the 2nd Iteration, we use: $\boxed{y_2 = y_1 + \Delta x \left.\frac{dy}{dx}\right|_{x=x_1}}$.

Note that each subsequent iteration uses the results from the previous one.

Given $\frac{dy}{dx} - 2x = \sin y^2$, and $y(2) = 4$, use Euler's Method and show your work in tabular form to find f(4.7) using 3 equal steps.

Solution :

$\frac{dy}{dx} = 2x + \sin y^2$ Also we know that the "Step Value", Δx is $\left(\frac{4.7 - 2}{3}\right) = 0.9$

x	y	y '	Δx	y-next
2	4	3.712096683	0.9	7.340887015
2.9	7.340887015	5.336849147	0.9	12.14405125
3.8	12.14405125	7.775953234	0.9	19.14240916
4.7	19.14240916			

$\boxed{y(4.7) \approx 19.14240916}$

AP Calculus BC 1 Assignment 148 Friday, December 18, 2015 Hour Name

Calculator Active

1. $y' = 3x - 2y$, $y(0) = 3$. Use Euler's Method to find an approximation for $y(0.05)$ using 2 equal steps.

2. $y' = x + y$, $y(2) = 3$. Use Euler's Method to find an approximation for $y(4)$ using a step size of 0.5.

3. $y' = e^{xy}$, $y(0) = 1$. Use Euler's Method to find an approximation for $y(0.3)$ using a step size of 0.1.

4. $y' = \cos x + \sin y$, $y(0) = 5$. Use Euler's Method to find an approximation for $y(0.4)$ using 3 equal steps.