

Recall that the  $n^{\text{th}}$  root of  $a$  can be represented as  $\sqrt[n]{a} = a^{1/n}$ .

Similarly,  $\sqrt[n]{a^b} = a^{b/n}$

To find  $8^{2/3}$ , we can consider  $\sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$

$$27^{4/3} = (\sqrt[3]{27})^4 = 3^4 = 81$$

$$49^{3/2} = 7^3 = 343$$

### Real $n^{\text{th}}$ Roots of $a$

Let  $n$  be an integer ( $n > 1$ ) and let  $a$  be a real number.

$n$ is an even integer.	$n$ is an odd integer.
$a < 0$ No real $n^{\text{th}}$ roots	$a < 0$ One real $n^{\text{th}}$ root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$ One real $n^{\text{th}}$ root: $\sqrt[n]{0} = 0$	$a = 0$ One real $n^{\text{th}}$ root: $\sqrt[n]{0} = 0$
$a > 0$ Two real $n^{\text{th}}$ roots: $\pm \sqrt[n]{a} = \pm a^{1/n}$	$a > 0$ One real $n^{\text{th}}$ root: $\sqrt[n]{a} = a^{1/n}$

Find all real cube roots of -216. There is **exactly 1 real solution** because 3 is odd and  $-216 < 0$ .

$$\sqrt[3]{-216} = -6$$

Find all real 4<sup>th</sup> roots of 81. There are **exactly 2 real solutions** because 4 is even and  $81 > 0$ .

Because  $3^4 = 81$  and  $(-3)^4 = 81$ , we can write  $\pm \sqrt[4]{81} = \pm 3$  or  $\pm 81^{1/4} = \pm 3$ .

Find the real solutions of the following

- The 4<sup>th</sup> root(s) of 16:  $\pm 2$
- The square root(s) of -49: Does not exist or DNE
- The cube root(s) of -125: 5
- The 5<sup>th</sup> root(s) of 243: 3

Evaluate  $16^{3/2}$

$$16^{3/2} = (16^{1/2})^3 = 4^3 = 64$$

$$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$$

Evaluate  $34^{-3/5}$

$$34^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(32^{1/5})^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(\sqrt[5]{32})^3} = \frac{1}{2^3} = \frac{1}{8}$$

Use a calculator to evaluate the following:

- $9^{1/5}$   $9^{(1/5)} = 1.552$
- $12^{3/8}$   $12^{(3/8)} = 2.539$
- $(\sqrt[4]{7})^3$   $7^{3/4} = 7^{(3/4)} = 4.304$

Find the indicated real  $n^{\text{th}}$  root(s) of  $a$ .

1.  $n = 3, a = 8$
2.  $n = 5, a = -1$
3.  $n = 2, a = 0$
4.  $n = 4, a = 256$
5.  $n = 5, a = -32$
6.  $n = 6, a = -729$

Evaluate the expression without using a calculator.

7.  $64^{1/6}$
8.  $81^{1/3}$
9.  $25^{3/2}$
10.  $81^{3/4}$
11.  $(-243)^{1/5}$
12.  $(-64)^{4/3}$
13.  $8^{-2/3}$
14.  $16^{-7/4}$

Evaluate the expression using a calculator. Round your answer to two decimal places when appropriate.

15.  $\sqrt[5]{32,768}$
16.  $\sqrt[7]{1685}$
17.  $25^{-1/3}$
18.  $85^{1/6}$
19.  $20,736^{4/5}$
20.  $86^{-5/6}$
21.  $(\sqrt[4]{187})^3$
22.  $(\sqrt[5]{-8})^8$