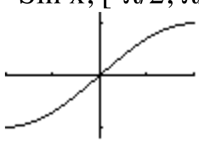
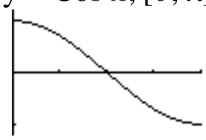
Pre-Calculus
146
Recall that A function has an Inverse iff it is one-to-one.
If we consider the function y = sin x, it cannot have an inverse beause it is not a one-to-one function over its entire domain of real numbers. But, if we restrict the domain to $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, then this restricted function is one-to-one. Often this new function is written as y = Sin x. The uppercase S indicates the restricted version.
In a similar fashion, we must restrict y = cos x, y = tan x, y = cot x, y = sec x, and y = csc x to insure that they are also one-to-one before seeking their inverses.

$\sin\left(\frac{\pi}{6}\right)=\frac{1}{2} \leftrightarrow \arcsin\left(\frac{1}{2}\right)=\frac{\pi}{6}$ $\quad \tan\left(\frac{\pi}{4}\right)=1 \leftrightarrow \tan^{-1}(1)=\frac{\pi}{4}$ $\quad$ arccos 2 $\rightarrow$ DNE

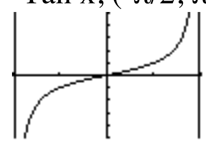
y = Sin x, [-π/2, π/2]



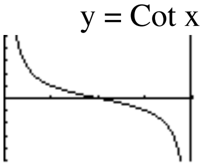
y = Cos x, [0, π]



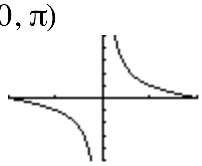
y = Tan x, (-π/2, π/2)



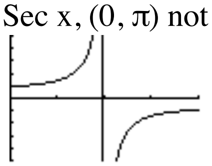
y = Cot x, (0, π)



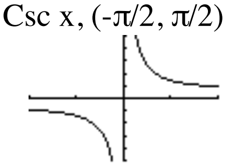
or



y = Sec x, (0, π) not π/2



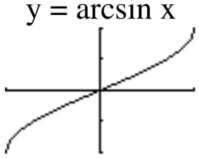
y = Csc x, (-π/2, π/2) not 0



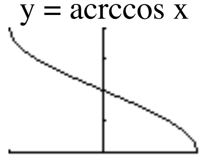
The Inverse for a Trigonometric Function is indicated by Adding the prefix "arc" or a superscript of -1 after the function. For instance, the Inverse of y = Sin x, is written as y = arcsin x or y = $\sin^{-1}$x or y = asin x.

Recall that The Graphs of Inverses are mirror images about the line y = x compaired to the original functions.

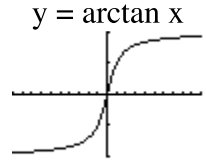
y = arcsin x



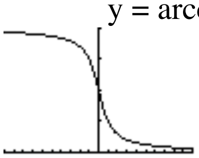
y = acrccos x



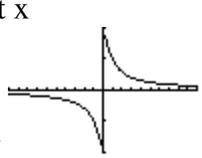
y = arctan x



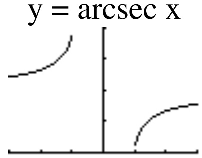
y = arccot x



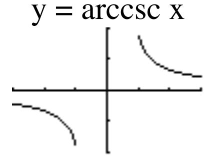
or



y = arcsec x



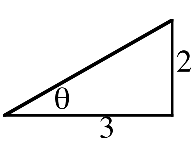
y = arccsc x

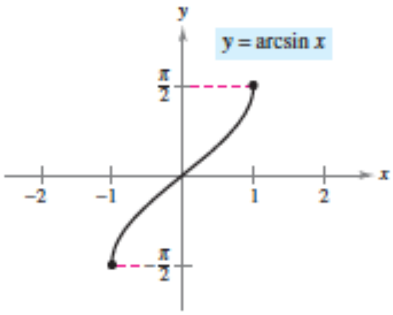


arccos(1/5) refers to the "Angle Whose Cosine is 1/5".

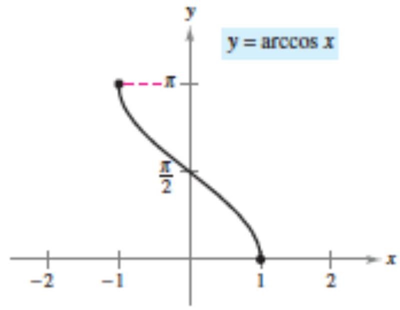
sin(arctan 2/3) means: Find the sin of the angle whose tangent is 2/3. Drawing a triangle with the tangent equal to 2/3 can be helpful.

In this figure the hypotenuse = $\sqrt{13}$. Therefore the sin of the angle = $\frac{2}{\sqrt{13}} = \boxed{\frac{2\sqrt{13}}{13}}$

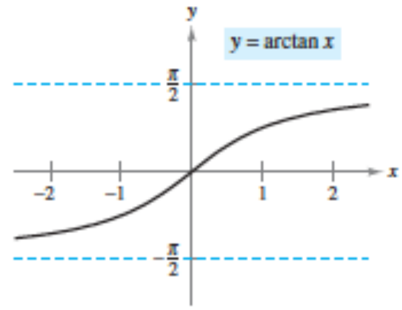




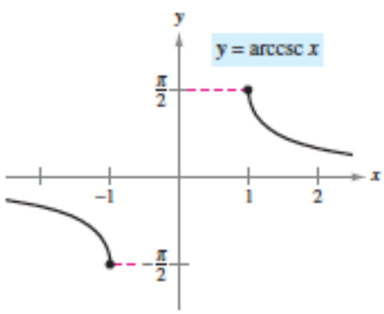

Domain: $[-1, 1]$
Range: $[-\pi/2, \pi/2]$



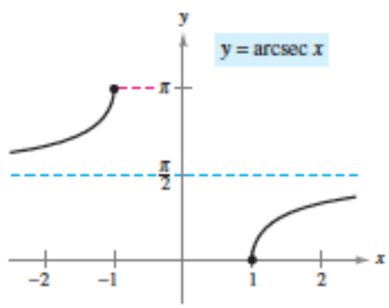

Domain: $[-1, 1]$
Range: $[0, \pi]$



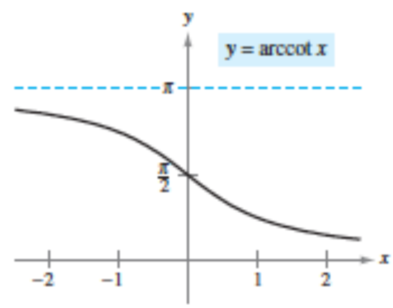

Domain: $(-\infty, \infty)$
Range: $(-\pi/2, \pi/2)$




Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[-\pi/2, 0) \cup (0, \pi/2]$




Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[0, \pi/2) \cup (\pi/2, \pi]$




Domain: $(-\infty, \infty)$
Range: $(0, \pi)$

Find $\arcsin\left(\sec\frac{\pi}{5}\right)$

*Solution*:

Since the Range (outcomes) of the secant is $(-\infty,-1]\cup[1,\infty)$ the $1<\sec\frac{\pi}{5}<\infty$.

The Domain of arcsin is [-1, 1].

Therefore $\sec\frac{\pi}{5}$ is outside the domain of arcsin.

$\arcsin\left(\sec\frac{\pi}{5}\right)$ Does Not Exist.

Pre-Calc 1 Assignment 146 Friday, December 11, 2015 Hour Name ____________________

NO CALCULATORS

| | |
|---|---|
| 1. $\arcsin\frac{1}{2}$ | 2. $\arctan\frac{\sqrt{3}}{3}$ |
| 3. $\arccos\left(-\frac{1}{2}\right)$ | 4. $\arctan\left(-\sqrt{3}\right)$ |
| 5. $\cos^{-1}\left(\frac{\pi}{3}\right)$ | 6. $\sin^{-1}(-1)$ |
| 7. arctan 1 | 8. arctan(-1) |
| 9. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ | 10. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ |
| 11. $\cos^{-1} 0$ | 12. arccot 0 |
| 13. arctan 0 | 14. arccos(-1) |
| 15. $\arcsin\left(\sin\frac{5\pi}{2}\right)$ | 16. $\cos^{-1}\left(\tan\frac{3\pi}{4}\right)$ |
| 17. $\tan\left[\arcsin\left(-\frac{3}{4}\right)\right]$ | 18. $\tan\left(\arccos\frac{x}{5}\right)$ |