AP Calculus BC
146

Solving Differential Equations – Separation of Variables:

When solving a separable differential equation, you MUST separate the variables. That is, everything pertaining to y must be on one side of the equation and everything pertaining to x must be on the other side of the equation. It is ABSOLUTELY NECESSARY to WRITE the equation this way in order to GET ANY CREDIT for this kind of problem.

Problem:
The Rate of change of y with respect to x is twice the product of x and y. When x is 0, then y is -5. Write the Specific equation for y as a function of x.

Solution:

$y' = 2xy$

$\frac{dy}{dx} = 2xy$

$\frac{dy}{y} = 2x\,dx$

$\int \frac{dy}{y} = 2\int x\,dx$

$\ln|y| = x^2 + C_1$

$|y| = e^{x^2 + C_1}$

$|y| = C_2 e^{x^2}$

Substitute the equation with (0, -5).

$|-5| = C_2 e^0 = C_2$

$C_2 = 5$

$|y| = 5e^{x^2}$

It may seem that $y = \pm 5e^{x^2}$

But $5e^{x^2}$ cannot be negative regardless of the value of x

And since y can be negative from the given information,

Therefore: $\boxed{y = -5e^{x^2}}$ is the Unique Solution Equation.

Problem:
The rate of change of V is proportional to V. When t = 0, V = 20,000 and when t = 4, V = 12,500. What is the value of V when t = 6?

Solution:

$\frac{dV}{dt} = kV$

$\frac{dV}{V} = k\,dt$

$\int \frac{dV}{V} = k\int dt$

$\ln|V| = kt + C_1$

$|V| = e^{kt + C_1}$ $|V| = C_2 e^{kt}$

$V = C_3 e^{kt}$

$20{,}000 = C_3 e^0$

$C_3 = 20{,}000$

$12{,}500 = 20{,}000e^{4k}$

$e^{4k} = \frac{5}{8}$

$4k = \ln\frac{5}{8}$

$k = \frac{1}{4}\ln\frac{5}{8}$

at t = 6, $V = 20{,}000e^{3/2 \ln(5/8)}$

$= 20{,}000\left(\frac{5}{8}\right)^{3/2} = \boxed{9{,}882.11768802618}$ Before Rounding.

AP Calculus BC 1 Assignment 146 Monday, December 14, 2015 Hour Name

Solve the differential equation.

1. $\frac{dy}{dx} = 4 - y$

2. $y' = \frac{5x}{y}$

3. $y' = \sqrt{x}\, y$

Write and solve the differential equation that models the verbal statement.

4. The rate of change of P with respect to t is proportional to 10 - t.

Write and solve the differential equation that models the verbal statement. Evaluate the solution at the specified value of the independent variable.

5. The rate of change of y is proportional to y. When x = 0, y = 4 and when x = 3, y = 10. What is the value of y when x = 6?

6. The rate of change of N is proportional to N. When t = 0, N = 250 and when t = 1, N = 400. What is the value of N when t = 4?