Algebra 2
146

Decartes's Rule of Signs:
Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ be a polynomial function with real coefficients.

* The number of positive real zeros of f is equal to the number of changes in sign of the coefficients of f(x) or is less than this by an even number.
* The number of negative real zeros of f is equal to the number of changes in sign of the coefficients of f(-x) or is less than this by an even number.

Example 1:
Find the number of possible positive real zeros, negative real zeros, and imaginary zeros for $f(x) = x^6 - 2x^5 + 3x^4 - 10x^3 - 6x^2 - 8x - 8$.

Solution:
The number of sign changes in $f(x) = x^6 - 2x^5 + 3x^4 - 10x^3 - 6x^2 - 8x - 8$ is 3
Therefore the possible number of positive real zeros is 3 or 1

The number of sign changes in $f(-x) = x^6 + 2x^5 + 3x^4 + 10x^3 - 6x^2 + 8x - 8$ is 3
Therefore the possible number of negative real zeros is 3 or 1

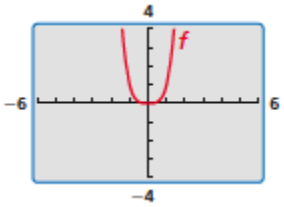
The Table below shows all the possibilities

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
3	3	0	6
3	1	2	6
1	3	2	6
1	1	4	6

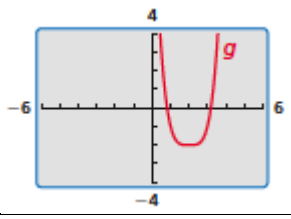
Example 2:
Test yourself and make a table of possibilities for zeros of $f(x) = 2x^6 - 5x^5 - 3x^4 + 3x^3 + 10x^2 + x - 9$.
Check your work with the following table.

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros
3	3	0	6
3	1	2	6
1	3	2	6
1	1	4	6

The graph below represents the function $f(x) = x^4$. Write the function for g(x) for each of a & b.

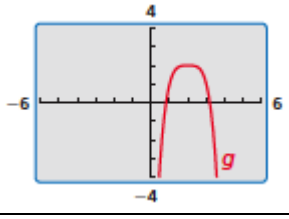


a.



$g(x) = (x - 2)^4 - 2$

b.



$g(x) = -(x - 2)^4 + 2$

Given $f(x) = 4x^3 - 5x^2 + 2x + 12$.
Find $g(x)$, a transformation of $f(x)$ by a reflection about the y-axis.

$g(x) = f(-x)$
$= 4(-x)^3 - 5(-x)^2 + 2(-x) + 12$
$= \boxed{-4x^3 - 5x^2 - 2x + 12}$

Given $f(x) = 3x^4 + 2x^3 - 5x^2 - 6x + 4$.
Find $g(x)$, a transformation of $f(x)$ by a reflection about the x-axis.

$g(x) = -f(x)$
$= -(3x^4 + 2x^3 - 5x^2 - 6x + 4)$
$= \boxed{-3x^4 - 2x^3 + 5x^2 + 6x - 4}$

Given $f(x) = x^3 + 5x^2 - 2x + 7$.
Find $g(x)$, a transformation of $f(x)$ by translation 2 units left and 3 units down.

$g(x) = f(x + 2) - 3$
$= (x + 2)^3 + 5(x + 2)^2 - 2(x + 2) + 7 - 3$
$= (x)^3 + 3(x)^2(2) + 3(x)(2)^2 + (2)^3 + 5(x^2 + 4x + 4) - 2x - 4 + 4$
$= x^3 + 6x^2 + 12x + 8 + 5x^2 + 20x + 20 - 2x$
$= \boxed{x^3 + 11x^2 + 30x + 28}$

Given $f(x) = 3x^2 - 2x + 4$.
Find $g(x)$, a translation of $f(x)$ by a reflection about the y-axis followed by a translation of 3 units left.
$g(x) = f(-x)$ followed by $f(-(x + 3)) = f(-x - 3)$
$f(-x) = 3(-x)^2 - 2(-x) + 4$
$= 3x^2 + 2x + 4$
$f(-(x + 3)) = 3(x + 3)^2 + 2(x + 3) + 4$
$= 3(x^2 + 6x + 9) + 2x + 6 + 4$
$= 3x^2 + 18x + 27 + 2x + 10$
$= \boxed{3x^2 + 20x + 37}$

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Complete the table for each function giving the number of possible kinds of zeros.

1. $g(x) = x^4 - x^2 - 6$

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros

2. $g(x) = -x^3 + 5x^2 + 12$

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros

3. $g(x) = x^3 - 4x^2 + 8x + 7$

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros

4. $g(x) = x^5 - 2x^3 - x^2 + 6$

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros

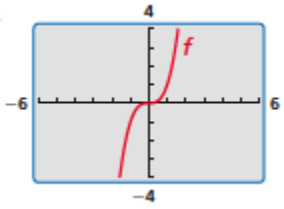
5. $g(x) = x^5 - 3x^3 + 8x - 10$

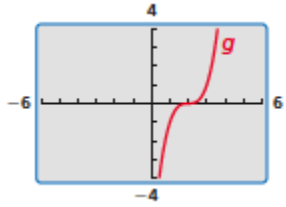
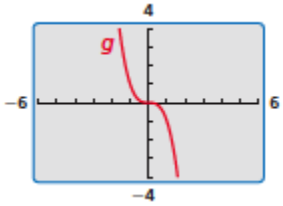
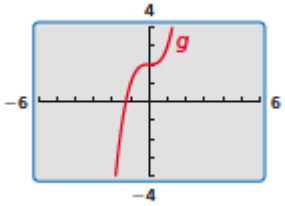
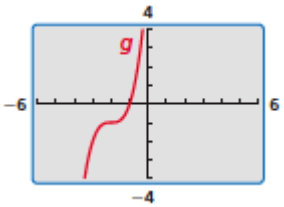
Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros

6. $g(x) = x^5 + 7x^4 - 4x^3 - 3x^2 + 9x - 15$

Positive Real Zeros	Negative Real Zeros	Imaginary Zeros	Total Zeros

7. The following graph represents the function: $f(x) = x^3$. Write the function for g(x) for each of a, b, c, & d.



a.	b.	c.	d.
			

Describe the following transformations of f represented by g.

8. $f(x) = x^5$; $g(x) = (x - 2)^5 - 1$

9. $f(x) = x^6$; $g(x) = -3x^6$

10. $f(x) = x^5$; $g(x) = \frac{3}{4}(x+4)^5$

11. $f(x) = x^4 + x^3 - 1$; $g(x) = f(-x) - 5$

Write the rule for g that represents the indicated transformations of the graph of f.

12. $f(x) = x^3 - 6$; translation 3 units left then a reflection in the y-axis.

13. $f(x) = x^3 + 2x^2 - 9$; Horizontal shrink by a factor of 1/3 and a translation 2 units up, followed by a reflection in the x axis.